

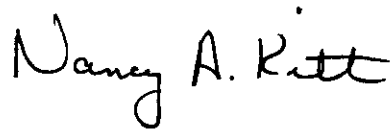
# Math Manipulatives as a Resource in the 3<sup>rd</sup> Grade Classroom

An Honors Thesis (HONRS 499)

by

Abigail L. Land

Thesis Advisor  
Ms. Nancy Kitt

A handwritten signature in black ink that reads "Nancy A. Kitt". The signature is written in a cursive style with a large initial 'N' and a small dot over the 'i' in 'Kitt'.

Ball State University  
Muncie, Indiana

December, 2003

Will Graduate May, 2004

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## Honors College Project Proposal

Name Abigail L. Land ID# 305-90-9457 E-mail alland@bsu.edu  
Address 101 North Martin Apartment #2 Phone (765) 741-0902  
Major(s) Elementary Education Graduation Date Spring 2004  
Academic Advisor Mrs. Gretchen Reid Project Advisor Miss Nancy Kitt  
Title of Honors Project Math Manipulatives as a Resource in The 3<sup>rd</sup> Grade Classroom

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1. The Honors project should broaden your educational experience through independent work that adds to your knowledge and develops your talents. How will your project help you accomplish these objectives and add to your personal goals? Why, in short, do you want to do this project?

I want to do this project because I know I will learn about the many different types of manipulatives that could be used in an elementary school. I will discover the many different ways these manipulatives can be used and the many different content areas of the Indiana State Standards they cover. I also think this would be an appropriate project because I will write lesson plans over many of these different manipulatives ideas, and these lesson plans could later be used in a third grade classroom.

2. When are you signing up for HONRS 499 (or what is the proposed equivalent)?

Fall 2003

3. What will be the outcome of this project? (Be specific-e.g., a paper, finished piece of research, creative activity...)

The outcome of this project will be a portfolio. The portfolio will contain information on the different manipulatives and their uses (divided into Indiana State Standard Content areas). Lesson plans will also be included in the portfolio to provide specific lesson information for all of the different third grade content areas.

4. Objective or Thesis (include target audience, purpose):

To create a portfolio that will help third grade teachers in the state of Indiana learn how to incorporate the use of manipulatives into their mathematics classroom in order to first teach students from a concrete level and to help those students who have trouble picturing what is being taught to them.

5. Project Description (please be specific- approximately two paragraphs):

The portfolio will be a three ring binder that is two inches wide and made to hold 8½ by 11 inch papers. In the very front of the portfolio there will be a title page, and following the title page will be a list of the resources used in the

construction of the portfolio. (The list of resources will be set up in the APA bibliography form.) The portfolio will then be divided into six sections using tabbed dividers. Each of these dividers will represent one of the Indiana State Standard content areas for third grade (number sense, computation, algebra and functions, geometry, measurement, and problem solving). Behind each of these dividers will be a copy of the Indiana State Standards from this area.

Included materials in each content area will include manipulative ideas and lesson plans. Each content area will contain photocopies of manipulative activities that could be done to cover the standards in that area. A lesson plan will also be included in each content area so an idea is given of how to turn one of the photocopied activities into an actual third grade classroom lesson. Finally, as an extra piece of information, a list of children's literature materials that could be used to cover that content area will be included at the end of the specific content area section.

6. State what the importance or implications of this project are. What do you expect to learn, or what would others learn from knowing about your anticipated results?

From this project I (as well as other teachers) will learn how many different ways manipulatives can be used in the third grade classroom and the many different Indiana State Standard content areas these manipulatives cover.

This project will be very beneficial because it is quite important that a student is first instructed from a concrete level using manipulatives so they can see the relationships as they complete their work hands-on. Then after this hands-on instruction, it will be much easier for the student to learn from a pictorial or abstract level. This project will assist teachers with their mathematics instruction because it provides many ideas for teaching from the concrete level using manipulatives.

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Student's Signature \_\_\_\_\_ Date \_\_\_\_\_

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Project Advisor's Signature\* \_\_\_\_\_ Date \_\_\_\_\_

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Honors College Approval \_\_\_\_\_ Date \_\_\_\_\_

\*Your signature indicates that you have read and approved this proposal.

## **Abstract and Acknowledgements**

### **Abstract**

In response to the Indiana Department of Education's desire to raise ISTEP scores in all public schools, mathematics is being very heavily stressed at the elementary school level. This has caused many schools to become quite involved in a drill and practice style of teaching mathematics. They believe it will best teach the students the information they need to know for the ISTEP. However, getting caught up in their desire to raise students' scores has lead many of these schools to forget most students learn best when they are first taught from a concrete or visual level, not by drill and practice alone. This can best be achieved in the elementary school classroom by using math related manipulatives to teach the students. In my thesis, I provide examples of different lesson plans and activities that teach students essential math concepts using math manipulatives. Each of these lessons and activities is tied directly to a third grade Indiana State Mathematics Standard and to a specific Indiana State Indicator that falls under that standard. This way third grade teachers are exposed to mathematics lessons that use hands-on manipulatives in order to cover the Indiana State Standards the teacher would be required to teach anyway.

### **Acknowledgements**

-I would like to thank Ms. Nancy Kitt for advising me as I worked on completing this creative project. She was extremely helpful throughout the entire process and I appreciate all the time and effort she put into assisting me along the way. Ms. Kitt is also responsible for instilling in me a love of using math manipulatives to improve instruction in the elementary school classroom.

-I would also like to thank those kind fellow Ball State University students, in particular one Miss Melissa Fritz, who spent many hours of their own time checking this project for errors in grammar, usage, and mechanics. I am truly grateful that you were all so willing to give of yourselves in order to help me.

## A Note From the Creator

I personally believe it is very important to teach mathematics using manipulatives in order to help students better understand the math concepts they are learning. I have used manipulatives in my teaching experiences and have witnessed first hand the benefits of using these hands-on materials. Students catch on to concepts faster when they first view them from a concrete or visual level with manipulatives (instead of just being thrown into workbook pages, which focus more on the abstract level of understanding). Students also seem to enjoy math more when they are taught with hands-on activities and allowed to explore the concepts in this medium first since. This is a result of the fact that students usually find these hands-on activities to be more exciting. Manipulatives can also be used one-on-one with students who have problems grasping the concepts you are trying to teach because they help students slow down and visualize what they are trying to solve. I personally believe manipulatives greatly improve the experience students have with mathematics in their elementary school years.

-Abby Land

Abby Land is a senior Elementary Education major who will graduate in the Spring of 2004 from Ball State University in Muncie, IN. Miss Land will also graduate with a History minor and an American Pluralism and Identity concentration in addition to her Elementary Education major. She is a member of the Ball State Honors College and will graduate with honors from the university. This thesis was completed as part of the Honors College requirements for graduation.

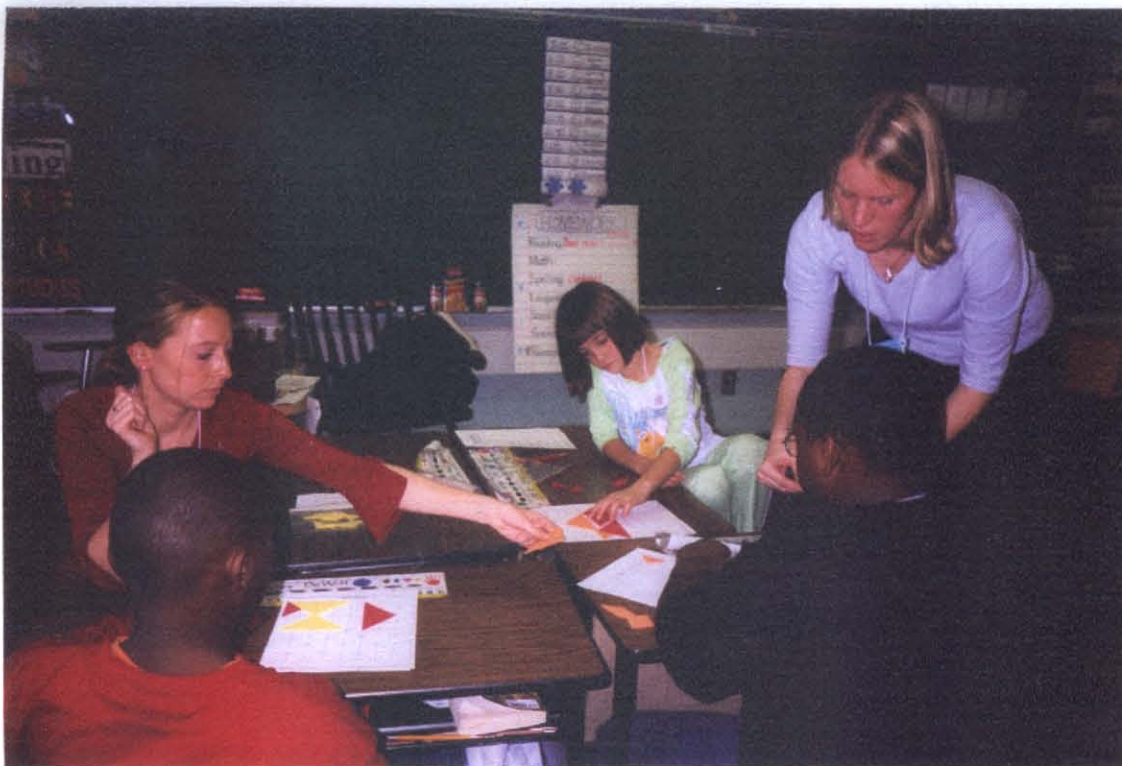


Miss Abby Land assists 3<sup>rd</sup> grade students as they use Fraction Islands for the first time.



Miss Land and a fellow teacher help the students model the correct fractions.





Miss Abby Land (blue shirt) and colleague help the students with their tangram game.



Miss Land and a fellow teacher explain a tangram assignment to the 3<sup>rd</sup> graders.

### **Standard 1 — Number Sense**

Understanding the number system is the basis of mathematics. Students extend their understanding of the place value system to count, read, and write numbers up to 1,000. They learn to order and round numbers up to 1,000. They develop the concept of equivalent fractions — fractions that look different, but have the same value — and use their understanding of equivalent fractions to compare the sizes of fractions. They also begin to develop the concept of decimals as a different way of representing fractional numbers.

## **Standard 1**

### **Number Sense**

*Students understand the relationships among numbers, quantities, and place value in whole numbers\* up to 1,000. They understand the relationship among whole numbers, simple fractions, and decimals.*

3.1.1 Count, read, and write whole numbers up to 1,000.

Example: Write 349 for the number “three hundred forty-nine”.

3.1.2 Identify and interpret place value in whole numbers up to 1,000.

Example: Understand that the 7 in 479 represents 7 tens or 70.

3.1.3 Use words, models, and expanded form to represent numbers up to 1,000.

Example: Recognize that  $492 = 400 + 90 + 2$ .

3.1.4 Identify any number up to 1,000 in various combinations of hundreds, tens, and ones.

Example: 325 can be written as 3 hundreds, 2 tens, and 5 ones, or as 2 hundreds, 12 tens, and 5 ones, etc.

3.1.5 Compare whole numbers up to 1,000 and arrange them in numerical order.

Example: What is the smallest whole number you can make using the digits 4, 9, and 1? Use each digit exactly once.

3.1.6 Round numbers less than 1,000 to the nearest ten and the nearest hundred.

Example: Round 548 to the nearest ten.

3.1.7 Identify odd and even numbers up to 1,000 and describe their characteristics.

Example: Find the even number: 47, 106, 357, 629.

3.1.8 Show equivalent fractions\* using equal parts.

Example: Draw pictures to show that  $\frac{3}{5}$ ,  $\frac{6}{10}$ , and  $\frac{9}{15}$  are equivalent fractions.

3.1.9 Identify and use correct names for numerators and denominators.

Example: In the fraction  $\frac{3}{5}$ , name the numerator and denominator.

3.1.10 Given a pair of fractions, decide which is larger or smaller by using objects or pictures.

Example: Is  $\frac{3}{4}$  of a medium pizza larger or smaller than  $\frac{1}{2}$  of a medium pizza? Explain your answer.

3.1.11 Given a set\* of objects or a picture, name and write a decimal to represent tenths and hundredths.

Example: You have a pile of 100 beans and 72 of them are lima beans. Write the decimal that represents lima beans as a part of the whole pile of beans.

3.1.12 Given a decimal for tenths, show it as a fraction using a place-value model.

Example: Show the decimal 0.7 as a fraction using pennies.

3.1.13 Interpret data displayed in a circle graph and answer questions about the situation.

Example: Have the students in your class choose the pizza they like best from these choices: cheese, sausage, pepperoni. Use a spreadsheet to enter the number of students who chose each kind and make a circle graph of the data. Determine the most popular and the least popular kind of pizza, and explain what the circle and each pie slice represent.

3.1.14 Identify whether everyday events are certain, likely, unlikely, or impossible.

Example: It is raining in your neighborhood. Is it certain, likely, unlikely, or impossible that the tree in your front yard will get wet?

3.1.15 Record the possible outcomes for a simple probability experiment.

Example: Predict how many heads and tails will occur if a coin is tossed 10 times. Have a partner toss a coin while you keep a tally of the outcomes. Exchange places with your partner and repeat the experiment. Explain your results to the class.

\* whole numbers: 0, 1, 2, 3, etc.

\* equivalent fractions: fractions with the same value (e.g., 1.2, 2.4, 3.6, etc.)

\* set: collection of objects, numbers, etc.

## Example Number Sense Lessons

## Lesson Plan Content Page

**Subject:** Math

**Lesson Topic:** Fractions (Equivalence and Comparison)

**INTASC Principle:** The professional educator understands content. (#1)

**IN State Standard:** Standard 1 Number Sense- Students understand the relationships among numbers, quantities, and place value in whole numbers up to 1,000. They understand the relationship among whole numbers, simple fractions, and decimals.

**IN State Indicator:** 3.1.8- Show equivalent fractions using equal parts.

### **Annotated Bibliography:**

#### **Instructional:**

Lee, Dennis. Alligator Pie. Toronto, ON: Macmillan Publishers, 1987.

This book will be used during the motivation section of the lesson to introduce the students to the concept that will be discussed in this lesson, fractions. The book is a great piece of instructional material because it provides picture graphs on all the pages. This way the students can see a visual representation of the fraction being discussed.

Robold, Alice I., Sandra L. Canter, and Nancy A. Kitt. Fraction Islands. Huntington, IN: Pathfinder Services, Inc., 1977.

This book provides many different ideas for activities that could be used to teach fraction equivalence. For my purposes I used the activities in Section 3 (Equivalent Fractions), lessons 1-3. This book is part of my own personal collection (it came as part of the Fraction Islands kit).

#### **Informational:**

Cathcart, George W., Yvonne M. Pothier, James H. Vance, Nadine S. Bezuk. Learning Mathematics: In Elementary and Middle Schools. Upper Saddle River, NJ: Merrill Prentice Hall (Pearson Education), 2003.

This book provides information on equivalent fractions, and it is where the definition of equivalent fractions that was used in this lesson was found. This book is a great source of information for people who are not familiar with many different basic math concepts. This book was used in my math methods course and it is now part of my own personal collection.

Kitt, Nancy A. "Math 391 Class Notes." Ball State University, Muncie, IN: 10/17 and 10/22/2002.

These notes provided many different pieces of information on fractions and fraction equivalence. The notes provided lots of helpful information on how to use Fraction Islands. The notes were taken in my math methods class with Ms. Nancy Kitt and are now part of my own personal collection of resources.

## Lesson Plan

**IN State Standard:** Standard 1 Number Sense- Students understand the relationships among numbers, quantities, and place value in whole numbers up to 1,000. They understand the relationship among whole numbers, simple fractions, and decimals.

**IN State Indicator:** 3.1.8- Show equivalent fractions using equal parts.

**Lesson Objective:** The students will be able to identify equivalent fractions using manipulatives after teacher instruction.

**Materials/Media:** Overhead, overhead dot paper (2 sheets), overhead markers (multicolor), Overhead Fraction Islands kit, Foamy Fraction Islands kits (one per student), dot paper sheets (2 for each student), pencils (one for each student), crayons/markers (each student should have their own set), *Alligator Pie*, worksheets (one for each student), student answer sheets (one for each student), and teacher answer sheet.

### New Information:

- An equivalent fraction is a fraction that is of equal value. Equivalent fractions are fractions equal too each other.
- Geoboards and fraction islands can be used to show fraction equivalence.

**Motivation:** The teacher will begin by reading the book *Alligator Pie* to the students. (This book is about two alligators that want to enjoy their pie, but more and more alligators keep showing up so the pie keeps getting divided into smaller sections. This book also provides pictures of the fraction pie on every page so the students can see the divisions occurring.) After reading the book the teacher will ask the students, “What happened to the fraction of the pie the two alligators got when the rest of the alligators kept showing up?” “What happened to the fraction of the pie the two alligators got at the very end when they ran away from the other alligators?” “Now that we have discussed this book about fractions can someone tell me what they already know about fractions?” “Have you ever used Fraction Islands to model fractions before?” (**Questions, Cues, Advance Organizers**) “Today we will be learning about equivalent fractions and also about comparing larger and smaller fractions using Fraction Islands.” **Goal for Learner**

### Procedure:

1. **New Information:** “Does anyone know what the word equivalent means?” (Encourage students to guess and predict the word meaning by using prior knowledge.) The teacher will then tell the students the definition of equivalent is equal value. This means equivalent fractions are fractions equal to one another. The teacher will pass out the Fraction Island kits to the students. While doing this the teacher will ask the students if they have worked with geoboards before. If they have worked with geoboards before, the teacher will ask them what they used the geoboards for, and if they have ever seen Fraction Islands.

2. **Modeling:** The teacher will make two  $3 \times 4$  units on the geoboard. Next the teacher will fill one unit with brown islands and the other unit with light blue islands. The teacher will then ask the students, "How many islands fill each region?" Then the teacher will remove one brown island. "What does the brown island left model?" (**Questions, Cues, Advance Organizers**) The teacher will tell the students the island on the geoboard models one-half if they do not answer correctly. The teacher will then explain the see over fill concept to the students (you see one brown it takes two brown to fill the unit, so the fraction is one over two).

The teacher will cover this brown island with light blue islands. "How many light blue islands exactly cover the brown island?" "What fraction are we modeling with the two light blue islands (remember to use see over fill)?" This proves one-half and two-fourths are equivalent fractions. The teacher will then demonstrate how to draw these pictures on the overhead dot paper. (**Non-linguistic Representations**)

The teacher will now cover the brown island exactly with red islands. "What fraction do the red islands represent (see over fill)?" The teacher will draw this on the dot paper and write one-half is also equal to three-sixths. Now the teacher will use yellow islands to find another fraction equal to one-half. The students will then be asked, "What fraction does this represent?" Then the teacher will model this final fraction on the dot paper.

3. **Guided Practice:** The students will each make four  $2 \times 4$  units on their geoboard. (As the students are completing all of the following activities the teacher will be working with the overhead fraction islands so that the students can check their work.) The teacher will instruct the students to fill one unit with pink islands, one with red, and one with yellow. The teacher will then tell the students model one half in the fourth unit. The students will then cover this island (the one half) with red islands and decide what fraction is modeled. The students will now cover the red islands with yellow islands and tell what fraction is modeled. "What did we just learn about these fractions?" The teacher will now have the students model these fractions on the dot paper. (**Non-linguistic Representations**)

The students will now create a  $3 \times 6$  unit on the geoboard, and model one-half. The teacher will let the students find out how many yellow islands cover this island and what fraction they represent. In a new  $3 \times 6$  unit, the teacher will have the students model two-thirds (see two islands, takes three to fill). "What could we use to cover these islands?" The teacher will instruct the students to find all the possible fractions that are equivalent to two-thirds (the pink, light blue, red, yellow). After modeling these on their geoboards the students will draw the representations on dot paper. (**Non-linguistic Representations**)

4. **Check for Understanding:** The teacher will have each student take out a piece of paper and ask them to write down the answer to this question, "What is an equivalent fraction?" (**Questions, Cues, Advance Organizers**)



The teacher will then give the students all time to answer the question. The teacher will do this instead of asking the question out loud, that way every student has time to think about the answer, not just the student who answers for the class. Then the teacher will instruct the students to share their answers with their group and to come up with one answer for the whole group. The teacher will then call on the students and each group will share their answer (this repetition should really help the students remember the definition).

5. **Practice/Application:** The teacher will pass out a worksheet to the students containing Fraction Island problems for them to complete on their own. The worksheet will be composed of the activities listed below.

Make a 4x6 unit on your geoboard, then make models for these explorations and draw the results on the dot paper. **(Non-linguistic Representations)**

- Show the island that models one-fourth.
- Cover this island with light blue islands. What fraction do these light blue islands model?
- Cover the island with red islands. What fraction do these red islands model?
- Cover the island with yellow islands. What fraction do these yellow island model?

Make a 3x5 unit on your geoboard for these explorations. After making the models on the geoboard draw the results on the dot paper and write the fraction numerals.

- Show the island that models one-third.
- Cover the island with yellow islands. What fraction is equivalent to one-third?
- Show the islands that model two-fifths. Cover these islands with yellow islands. What fraction is shown to be equivalent to two-fifths?

6. **Closure:** The teacher will ask the students, “What did you learn during today’s lesson?” The teacher and the students will discuss the answers. Then the teacher will ask the students, “How did we use the geoboards to create equivalent fractions?” “How can you tell if something is an equivalent fraction?” “What is the definition of an equivalent fraction?” The teacher will have the students respond to each of these questions after they are asked. **(Questions, Cues, Advance Organizers)**

**Evaluation of Student Learning:** The teacher will collect the worksheets the students complete during the practice/application section of the lesson. The teacher will then use an answer sheet to grade the student responses. (This will allow the teacher to see which students understood the lesson and which students still need more help.)

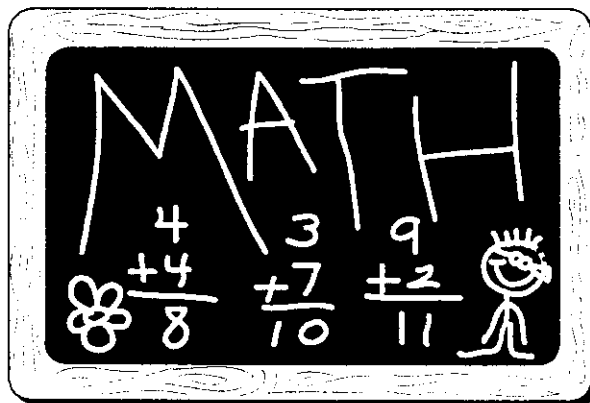
**Lesson Extension:** If there is time left over, or if the students catch on to the lesson very quickly and need more of a challenge, the teacher will begin to use the Fraction Islands not only to discuss the concept of equivalence, but also to teach the students how to

determine which fractions are larger or smaller using comparison. The teacher will do this by having the student create two units that are the same size on the geoboard. The teacher will then have the students model one half in one unit and two thirds in the other. The teacher will then have the students explain which one is larger and why. The lesson will continue on in this manner.

## Instructions

- A. Make a 4x6 unit on your geoboard, then make models for these explorations and draw the results on the dot paper.
1. Show the island that models one-fourth.
  2. Cover this island with light blue islands. What fraction do these light blue islands model?
  3. Cover the island with red islands. What fraction do these red islands model?
  4. Cover the island with yellow islands. What fraction do these yellow island model?
- B. Make a 3x5 unit on your geoboard for these explorations. After making the models on the geoboard draw the results on the dot paper and write the fraction numerals.
1. Show the island that models one-third.
  2. Cover the island with yellow islands. What fraction is equivalent to one-third?
  3. Show the islands that model two-fifths.
  4. Cover these islands with yellow islands. What fraction is shown to be equivalent to two-fifths?

\*Be sure to write the fraction that the picture represents on the line, and draw the picture of the fraction on the dot paper.



NAME \_\_\_\_\_

A.

1.

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2.

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3.

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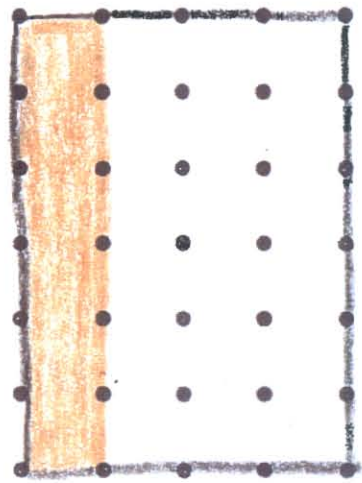
B. 1. \_\_\_\_\_ 2. \_\_\_\_\_

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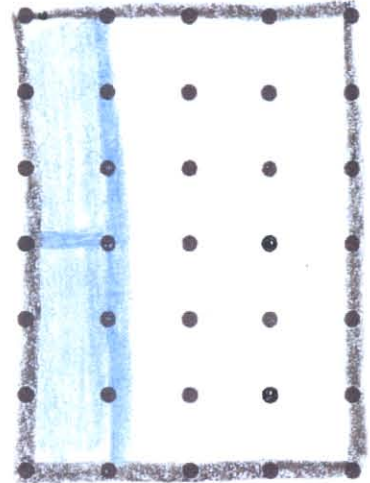
3. \_\_\_\_\_ 4. \_\_\_\_\_

\_\_\_\_\_

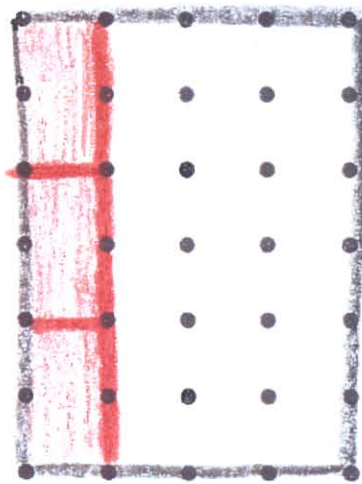
A. 1.  $\frac{1}{4}$



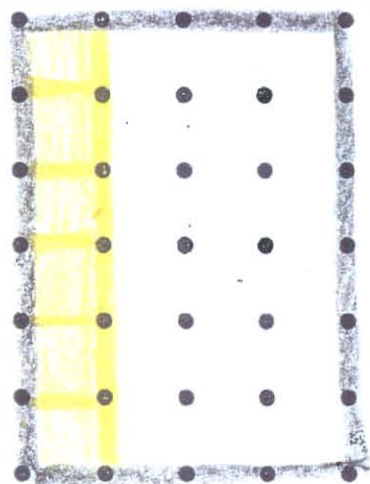
2.  $\frac{2}{8}$



3.  $\frac{3}{12}$



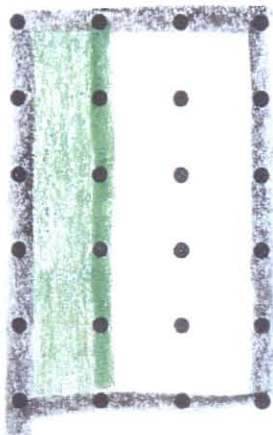
4.  $\frac{6}{24}$



B.

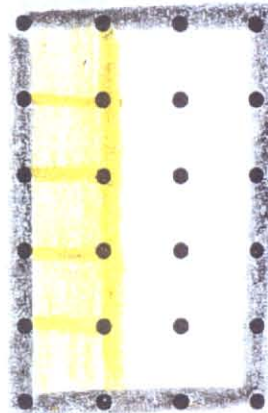
1.

$$\frac{1}{3}$$



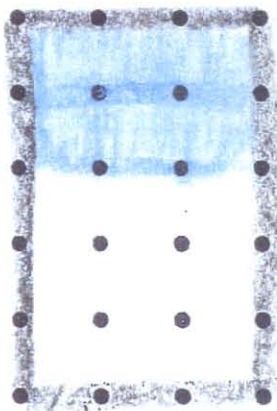
2.

$$\frac{5}{15}$$



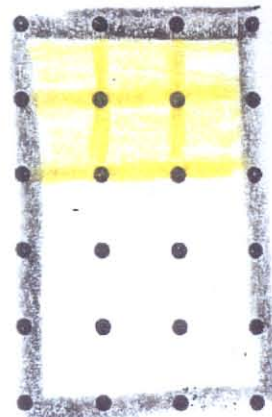
3.

$$\frac{2}{5}$$



4.

$$\frac{6}{15}$$



## **Lesson Plan Content Page**

**Subject:** Math

**Lesson Topic:** Probability (Casino Math)

**INTASC Principle:** The professional educator understands content. (#1)

**IN State Standard:** Standard 1 Number Sense- Students understand the relationship among numbers, quantities, and place value in whole numbers up to 1,000. They understand the relationship among whole numbers, simple fractions, and decimals.

**IN State Indicator:** 3.1.14- Identify whether everyday events are certain, likely, unlikely, or impossible.

3.1.15- Record the possible outcomes for a simple probability experiment.

### **Annotated Bibliography:**

#### **Instructional:**

Cathcart, George W., Yvonne M. Pothier, James H. Vance, Nadine S. Bezuk. Learning Mathematics: In Elementary and Middle Schools. Upper Saddle River, NJ: Merrill Prentice Hall (Pearson Education), 2003.

This book provides an activity on page 375 that was used in this lesson, and shows the types of teacher made charts that were used. The Cathcart book was my textbook for my Math 391 class, and is now part of my own personal collection.

#### **Informational:**

Kitt, Nancy A. "Math 391 Class Notes." Ball State University, Muncie, IN: 11/26/2002.

These notes provided information on the two different types of probability, experimental and theoretical, that I as a teacher needed to learn about before I could teach this lesson. The notes were taken in my math methods class with Ms. Nancy Kitt, and are now part of my own personal collection of resources.

Reesink, Carol J. Teacher-Made Aids for Elementary School Mathematics: Volume Two. Virginia: The National Council of Teachers of Mathematics, Inc., 1985.

This book explains how to make spinners that can be used when teaching probability lessons in the classroom. This book is part of Ms. Nancy Kitt's own personal collection, and can be found in her office on the fourth floor of the Robert Bell Building.



## Lesson Plan

**IN State Standard:** Standard 1 Number Sense- Students understand the relationship among numbers, quantities, and place value in whole numbers up to 1,000. They understand the relationship among whole numbers, simple fractions, and decimals.

**IN State Indicator :** 3.1.14- Identify whether everyday events are certain, likely, unlikely, or impossible.

3.1.15- Record the possible outcomes for a simple probability experiment.

**Lesson Objective:** The students will determine the probability of events by participating in Las Vegas spinner and dice games.

**Materials/Media:** Lesson plan, bag of 15 colored balls, two spinners, a set of dice for every set of two students (half red and half white), probability charts (one for each pair of students), probability tables (one for each pair of students), and a deck of cards.

### New Information:

- Probability is the chance that an event, something that brings about an outcome, will occur.
- A number written as a fraction can be used to determine the probability.
- Experimental probability comes from performing an experiment; it is not always the same as the theoretical probability, which is the mathematical calculation of what the probability should be.
- (Theoretical) Probability =  $\frac{\text{number of ways the particular event happens}}{\text{number of ways that all events can happen}}$

**Motivation:** The teacher will be dressed up in casino like attire to set the mood for the students. (For example: Black pants, white shirt, and a green visor.)

Lay out one of each of the three different colors of bouncy balls on the work area. Tell the students, "I have 15 bouncy balls in this bag and the bouncy balls are all three of these colors. You do not know how many bouncy balls of each color are in my bag, therefore, we will need to do an activity to figure it out." Have the class predict how many balls of each color are in the bag. Once the group has come to a unanimous decision, begin pulling balls out of the bag one at a time without looking inside. After the third ball has been pulled out of the bag, give the students a chance to change their prediction if they so desire. **(Generating/Testing Hypotheses)** Do this three times and discuss the findings with the class. Discuss the likeliness of pulling out a certain color, and the certainty that each color could be pulled out. Point out that none of the colors are impossible to pull out of the bag. Reveal how many of each color ball were in the bag and see how close the prediction was, discuss how the prediction got closer every time three more balls were pulled out of the bag. "Today we are going to learn about the probability of events taking place in Las Vegas spinner and dice games." **Goal for Learner**

**Procedure:**

1. **New Information:** “Does anyone know what the word ‘probability’ means, or have an explanation of probability they want to share with the class?”  
**(Questions, Cues, Advance Organizers)** Allow the students time to discuss their definitions and ideas. Then explain to the students that probability is the chance an event will occur. Probability can be expressed as the chance a particular event will happen over the chance that all events will happen. Write this formula on the board. Then explain, if we wanted to see what the probability of drawing a red ball was, and there were 5 red balls out of the 15 balls total the probability then would be  $5/15$  or  $1/3$ .
2. **Modeling:** Take out a spinner that is equal parts red, blue, and white. Discuss the probability that the spinner will have an equal chance of landing on any of the colors. Have a student spin the spinner and have the class find the probability for the color they just spun ( $1/3$ ). “How could we increase our chances of winning?” Pull out another spinner with the red portion  $\frac{1}{2}$  of the total spinner, and the blue and white only  $\frac{1}{4}$ . “What do you think the probability is of getting a red? Is this better than last time?” **(Questions, Cues, Advance Organizers)** Have the students sit in a circle on the floor. Have each student spin the spinner and say the color out loud to the teacher who will record it on the board. Then have the student pass the spinner to the next person. Give every student a chance to spin the spinner and then tally the results. “Did the results come out the way we planned? Which color would you choose if you were in Vegas?” Explain to the students, “This is the experimental probability, a probability found by conducting experiments. The probability we found by using the formula is what the probability should be theoretically, or the theoretical probability.”
3. **Guided Practice:** Put the spinners away and take out one die. Discuss the probability of rolling each number using one die (it is equal). Model this if you need to by using the formula on the board. “What is the probability of rolling a five?” There is one five out of six possibilities so the probability is  $1/6$ . “What is the probability of rolling a 1?” There is one number one out of six total numbers, so the probability is  $1/6$ . Take out another die and ask the students, “What are the possible number outcomes now that there are two dice?” **(Questions, Cues, Advance Organizers)** The numbers that can be rolled when the two dice are added together are numbers 2-12. “What do you think it means to have an impossible probability? The number one now has an impossible probability. An impossible probability occurs when there is no chance that the event can occur. For example, with two dice there is no way that a number one can be rolled.”
4. **Check for Understanding:** “What is the definition of probability and how do you calculate theoretical probability? What is the difference between theoretical probability and experimental probability?” **(Questions, Cues, Advance Organizers)** Allow the class time to answer.

5. **Practice/Application:** Now divide the students into pairs and give each group of students one white and one red die and the probability table. The students will then be told that they will roll the pair of dice 36 times and that they will record this by placing tally marks in the top frequency section of the chart. **(Non-linguistic Representations)** “What number do you think you will roll the most? Why?” After the students have rolled and tallied 36 times have them count up the number of tallies in each column and write that number as the top number of the experimental probability fraction. “What numbers did you roll the most? Was this what you expected?” “Now we are going to complete a dice chart to find the theoretical probability (what should happen mathematically) instead of the experimental probability we found earlier.” Explain to the students how to fill in the dice chart. **(Non-linguistic Representations)** Once the chart is completed, have the students tally each outcome in the bottom frequency column. Then have the students count up the tally marks and transfer the number to the numerator of the theoretical fraction. Explain to the students that this is the mathematical chance of rolling a certain number and the more times you roll the dice, the closer you will get to this number.
6. **Closure:** “Why is the lucky number seven called the lucky number seven? How can understanding probability help you outside of school?” Have the class discuss both of these questions. If they cannot think of answers for the second question, mention things like winning board games or predicting card tricks.

**Evaluation of Student Learning:** The teacher will ask the class probability questions to test their knowledge. Such as: “If I put every student’s name in a hat, what is the probability I would draw out a boy’s name? What is the probability I would draw out a name that begins with a ‘K’? What is the chance I would draw out \_\_\_\_\_’s name?” More questions can be added as the teacher sees fit in order to test the students understanding of the concept.

**Lesson Extension:** Using a deck of cards, have the students find the probability of drawing a certain color card, a certain suit, a certain number, and finally an exact card. The students will discuss the probability of these answers and the teacher will ask, “are there any cards you are more likely to draw than the others?”



# Probability Chart

Red Dice

6						
5						
4						
3		5				
2						
1	2					
	1	2	3	4	5	6

White Dice

## More Teaching Ideas for Number Sense

## Teaching Ideas Bibliography (Number Sense)

Bamberger, Honi, and Patricia Hughes. Super Graphs, Venns, and Glyphs: Hundreds of Great Data Collecting Activities to Build Real-Life Math Skills. New York: Scholastic Professional Books, 1995.

\*Contains the "Circle Graphs" idea on pages 23 and 24.

Bloomer, Anne M., and Phyllis A. T. Carlson. Activity Math: Using Manipulatives in the Classroom. Menlo Park, CA: Addison-Wesley Publishing Company, 1993.

\*Contains "Place Value 4" and "Place Value 5" on pages 73, 74, 75, and 76.

Greenburg, Dan. Funny and Fabulous Fraction Stories: 30 Reproducible Math Tales and Problems to Reinforce Important Fraction Skills. New York: Scholastic Professional Books, 1996.

\*Contains the "Dear Ms. Fraction" story on pages 16 and 17.

Jones, Graham A., and Carol A. Thornton. Data, Chance, and Probability: Grades 4-6 Activity Book. Lincolnshire, IL: Learning Resources, Inc., 1993.

\*Contains the "Take a Candy", and "Candy Chances" activities on pages 38 and 39.

## Place Value 4

**Skill:** To understand place value through 999: regrouping and expanded notation

**Time:** 2 periods

### Materials:

*counters*

*crayons to match colors of counters*

*place value (pv) materials to 100s*

*adding-machine tape*

*3 dice*

See Reproducibles for: pv mats to 100s (later you can use pv mats to 1000s), number lines to 1000

### Anticipatory Set

Use *pv mats*, *counters*, and *crayons*.

Students color the pv mats to correspond to the selected value of each color of the counters. For example, if the ones place is to be shown with red counters, color the top of that column red.

Call out a number from zero to nine. Each student decides in which column to place that number. They place the counters on their mats.

Call out another number, and the students place that number. The third number called is placed in the remaining space.

Students compare numbers to see who has the largest number. How many different numbers did the class make?

### Procedure 1

To develop the concept of place value through 999

Use *pv mats* from *Anticipatory Set* and *pv materials*.

**Instruct students to place five cubes in the ones place. Have them chorally read "zero hundreds, zero tens, and five ones."**

**Now have them place four rods in the tens place and four more cubes in the ones place. Ask them to chorally read the number.**

Students respond: Zero hundreds, four tens, and nine ones.

**Ask: What will happen if you add one more cube to the ones place?**

Students respond: We will need to trade for another ten.

**Now have students place five rods in the tens place and one more cube in the ones place. Tell them to be ready to chorally read after trading up as needed.**

Students respond: One hundred, zero tens, zero ones.

Continue with the exercise until you have added blocks and read up to the number 999.

**Ask what will happen if we now add one more cube to the ones place. Discuss the need to have another place in order to create the number 1,000.**

### Procedure 2

To practice regrouping and expanded notation

Use *mats* from *Anticipatory Set*, *counters*, *adding-machine tape*, and *dice*.

**Roll dice and have students add the numbers together. Students place counters on their mats representing the total. Tell them to chorally read the number.**

Students respond: Zero hundreds, \_\_\_ tens, and \_\_\_ ones.

Each group assigns to one student the responsibility of writing the number on their adding-machine tape.

**Roll the dice again and follow the same procedure, reminding the students to trade up as needed and to be ready to chorally read the resulting number.**

Students do the assigned task and chorally read the resulting numbers. The assigned student writes this number on the adding-machine tape, making a number line.

Students are not all writing but are adding with you as an activity of the entire class, while the assigned students write on the adding-machine tape.

**Continue with this procedure until you reach a number above 900. If desired, continue until 999, being careful not to go over 999.**

**Have class orally read the numbers from the tape in unison, starting at the smallest and reading up to the final, or largest, number.**

Discuss the difference between reading chorally \_\_\_ hundreds, \_\_\_ tens, and \_\_\_ ones, and reading the numbers in the usual way.



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### Procedure 3



To write numbers through 999

Use *number lines* and *dice*.

Have students take the number lines and number from 100 to 1,000 on the long tic marks.

Roll the dice and read the numbers shown as a 3-digit number. Students write this number on the number line.

Continue until there are at least ten different numbers on the number line. Have a student read the numbers on the line from least to greatest. Do the other students agree with the order in which the numbers were read? Another student can read the numbers in expanded form.

After doing these activities, students discuss what they have learned about place value.

Discuss with students the kind of math they used to do the regrouping. How do they think these activities are connected?

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### Practice and Extension

Have students make up games to play in groups of two or three using *py mats to 1000s*, *cards with numbers zero to nine written on them*, and *counters*. They may need to color-code these mats as they did the earlier mats. These games can be left out to be used in free time to build competence. Some game ideas are Play to 1,000, Highest Number Wins, and Lowest Number Wins.

Have students display the *adding-machine-tape number lines from Procedure 2* around the school. Have students post a sheet under each line headed with a type of object such as paper clips. Other students can write problem situations on the paper, using the objects and one of the numbers shown. For example, how much would 742 paper clips weigh? How long would 321 paper clips be? Students can then estimate and measure.

# Place Value 5

**Skill:** To compare numbers through 999

**Time:** 1 period

**Materials:**

*deck of cards*

*paper:* cut into small slips

*counters:* color-coded to represent numbers to 100s

*dice:* 1 per group of three students

See Reproducibles for: place value (pv) mats to 100s or 1000s, number lines to 100—1 per student, bills to 100s—in different colors

## Anticipatory Set

Use *bills*, *pv mats*, and *cards*.

Draw a card and have students place that amount in the ones place. The next number is placed in the tens place; the third in the hundreds place.

How many \$1 bills does each \$100 bill stand for? (100) How many \$1 bills does each \$10 bill stand for? (10) Review reading numbers. Repeat until students can place and read numbers easily.

In their responses, students should give examples of items that might cost the different amounts of money. Students write the different amounts of money on slips of paper, one number per slip.

**Repeat with two new sets of three cards. Students place, then write their number, then order and write the numbers in their group. When they feel ready, they may write first, then check with the bills.**

**Have students take number lines and place the numbers in order on the line in the correct sequence.**

## Procedure 1

To develop the concept of comparing numbers through 999

Use *bills*, *pv mats*, *number lines*, *cards*, and *paper slips*.

**Draw a set of three cards and have students place and read the number as in the Anticipatory Set.**

Students leave this number on their mats.

**Rearrange the digits and have students place this number below the first.**

**Which number do they think is larger? Why is one number bigger, even though they both have the same three digits?**

Students discuss that the value of a number depends on its place.

**Repeat with three new cards.**

**Ask students to name a rule for comparing numbers.**

Students state that you start with the largest place and compare those numbers.

**Next draw two sets of three cards and turn them over. Have half of the students arrange bills for one of the sets in any order on their pv mats. The other half does the same for the second set.**

Students complete the activity. They should have many different amounts of bills on their mats.

**Discuss which amount of money is the greatest. The least? If they had items costing these various amounts of money, which would be the least expensive? Why? The most expensive?**

## Procedure 2

To compare numbers using addition and regrouping

Use *cards*, *pv mats*, and *counters*.

If students have not yet colored the top section of their pv mats, have them do so now. Be sure the colors used correspond to the colors of the counters, and that all students represent the same place with the same color.

Take the deck of cards, shuffle it, draw three cards, and call out the numbers, giving the face cards the value of zero. If a ten is drawn, ask students what they will need to do during that turn. (regroup)

Students place counters on their mats with the number of the largest numerical value being placed in the ones place and the next largest in the tens place. The number having the smallest numerical value should be put in the hundreds place.

Write the number on the board and ask for two other numbers that could be made using these digits.

Which is the least? The greatest? Can students order the numbers?

Continue, adding the new number onto the number on the mat until you reach a number in the 900s.

Repeat, having students write the numbers in order after placing counters to represent them. Continue having students both place counters and write numbers until they are competent.

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### Procedure 3



To practice comparing numbers

Use *dice*, *pv mats*, and *counters*.

Students work in groups of three. One student rolls a die. The other two students place that many counters on *pv mats* in whichever place they choose, and write their numbers. Counters *cannot be repositioned*. After three rolls, students read numbers; the student with the largest number wins that round.

Students continue the activity for about 10 minutes, rotating roles. At the end of about 5 minutes, change the rule so that the student with the smallest number wins the round.

**Have students discuss how their strategies changed when the rule was changed.**\_\_\_\_\_

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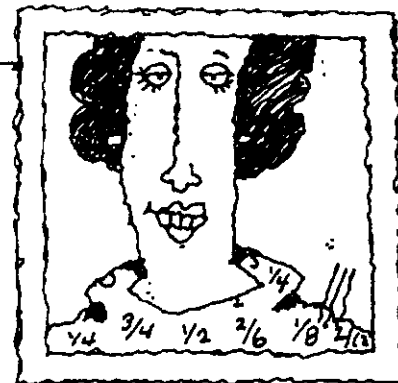
### Practice and Extension

Students use the *slips of paper with money amounts from Procedure 1*. They shuffle the slips of paper and order them from least to greatest. Another student can check. Have students make up problems listing objects that might cost these amounts of money. They then can order the items from most to least expensive.

Students continue the *game from Procedure 3*. They can play with a *timer* and change the rules every three minutes.

Name \_\_\_\_\_

# Dear Ms. Fraction



1.

Dear Ms. Fraction,

There's this really cute boy in school. I'll call him Randy. He offered me  $\frac{2}{3}$  of his candy bar.

Then there's this other really cute boy, Sandy. He offered me  $\frac{2}{5}$  of his candy bar.

So now I don't know. Who do you think really likes me more, Randy or Sandy?

Signed,  
CLUELESS IN SEATTLE

Dear Clueless,

*I don't know who likes you more, but I can tell you who is willing to give you more candy. Look at these two candy bars.*

**Color in  $\frac{2}{3}$  of the top one and  $\frac{2}{5}$  of the bottom one. Which fraction of candy bar is bigger?**

--	--	--

Randy's Candy Bar

--	--	--	--	--

Sandy's Candy Bar

Yours Truly,  
MS. FRACTION

2.

Dear Ms. Fraction,

This girl in my class, Jennifer, really thinks I'm cool. At least she thought I was cool. Then she offered to split a Mango Fango fruit drink with me. Well, Ms. Fraction, I forgot what I was doing and drank  $\frac{5}{6}$  of the Mango Fango. That made Jennifer mad. She says I'm worse than her last boyfriend, Lloyd, who drank  $\frac{7}{8}$  of a Choco-Rocko shake they were sharing.

So who's worse, Lloyd or me?

Signed,  
The Kansas City GULPer

Dear KCG,

Why don't you line up a Mango Fango next to a Choco-Rocko? **Color in  $\frac{5}{6}$  of the Mango Fango and  $\frac{7}{8}$  of the Choco-Rocko.** That should tell you who's the bigger chugger.

Yours Truly,  
MS. FRACTION


Mango  
Fango


Choco-  
Rocko

3.

Dear Ms. Fraction,

Help! I'm totally uncool. The way I see it, a cool baseball cap would make me  $\frac{1}{4}$  cooler than I am now. But a totally cool haircut would make me  $\frac{3}{8}$  cooler.

So which should I get, the hat or the hair?

Signed,  
Uncool in UPPER ONTARIO

Dear Uncool,

Personally, I think ice cubes in your shoes would keep you the coolest. But what do I know? **To compare the two cooling methods, divide these bars and see which fraction is bigger.**

$\frac{1}{4}$  cooler

$\frac{3}{8}$  cooler

Yours Truly,  
MS. FRACTION

4.

Dear Ms. Fraction,

I took a survey in school to find out what girls really like in a guy. I found that  $\frac{2}{3}$  of the girls liked a guy's smile,  $\frac{3}{4}$  thought a sense of humor was important,  $\frac{7}{8}$  liked hair best, and  $\frac{1}{7}$  felt that expensive shoes were the only thing that mattered. How can I make sense of these results?

Signed,  
Survey Sam

Dear Sam,

**Draw the fractions below on candy bars, pies, or whatever you like. Then list them in order from greatest to smallest. And if you want my opinion, it's no contest: It's the shoes.**

Yours Truly,  
MS. FRACTION

$\frac{2}{3}$

$\frac{3}{4}$

$\frac{7}{8}$

$\frac{1}{7}$

Greatest to Smallest:

\_\_\_\_\_

# Circle Graphs

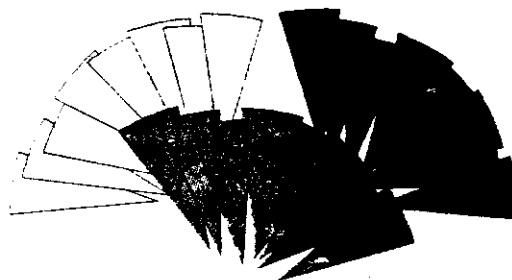
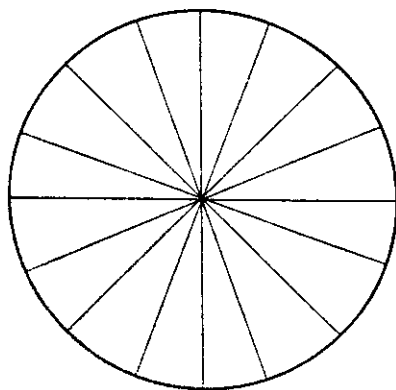
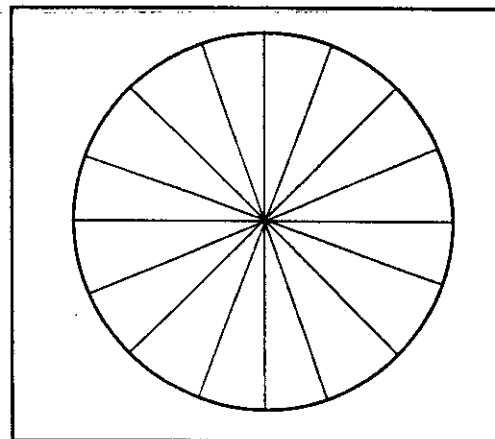
Circle graphs are quite popular in newspapers and magazines. A circle graph is another way to collect and represent data.

## MATERIALS

- assorted sheets of butcher paper (four different colors)
- scissors
- wipe-off markers
- heavy tagboard
- rubber cement or gluestick
- compass or protractor
- optional: laminating machine

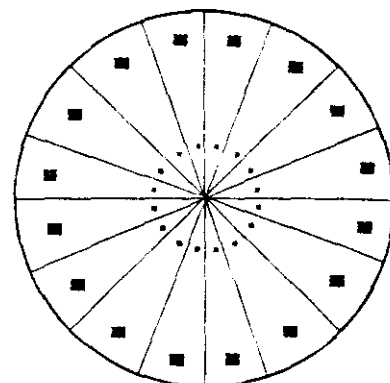
## DIRECTIONS

1. Using markers and a compass or protractor, draw a large circle on the butcher paper. Divide the circle into an even number of parts. If you have an odd number of people, add one more part. (Example: 27 people, divide circle into 28 parts.)
2. Cut around the outside of the circle only. Leave the inside pieces intact. This will be the circle graph on which children will place their votes.
3. Using the other three colored pieces of paper, make three circles. Cut around these circles and then cut the pieces apart. You will have three sets of circle parts.



4. Use the rubber cement to paste tagboard to the circle pieces. (The paper pieces will not hold up for long without the tagboard.) If possible, laminate the basic graph as well as all the pieces.

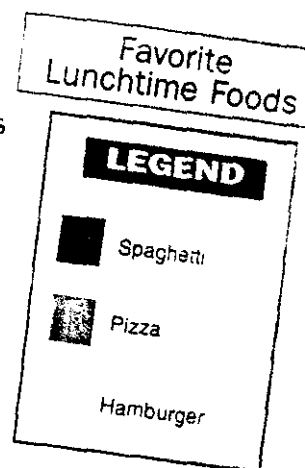
5. Lay the circle graph out in front of you. Cut Velcro into small pieces. Place one side of Velcro (loop side) on each piece on the graph. You may need to use two small pieces of Velcro—one for the narrower section, the other for the wider section of the circle. You will need more of the sticky side of the Velcro than the loop. Be prepared—you'll have leftovers!



6. Now place the other side of the Velcro (sticky side) onto your circle pieces. You will need to place the Velcro on the same part of the piece as the circle graph. Try to come as close as possible. Keep the Velcro in the same place for all the pieces. This part takes patience!

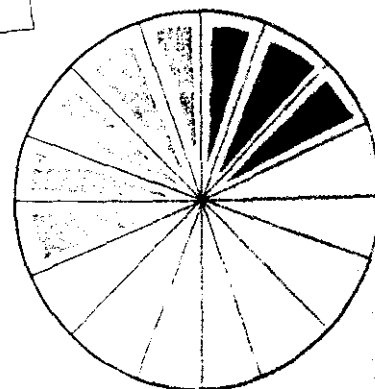
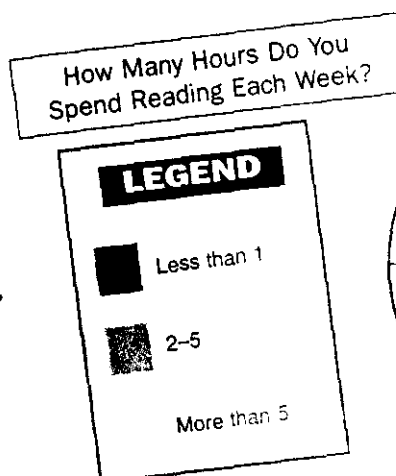


7. Design a legend for the graph using the three colored pieces. This is where your choices can be written. For example, for a circle graph on Favorite Lunchtime Foods, blue means "I like spaghetti," green means "I like pizza," purple means "I like hamburgers." Laminate, if possible.

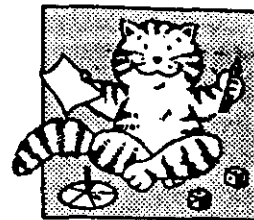


### SUGGESTED TOPICS

- Which school lunch would you rather eat? Meatballs and spaghetti, fish sticks, pizza.
- How many hours do you spend reading each week? Less than 1, 2-5, more than 5.
- Where would you rather spend your summer vacation? Home, camp, farm, other.



# Take a Candy



Name \_\_\_\_\_



A Mix of:

- 2 red
- 2 yellow
- 2 purple

When you open the candies, which color will you find first?

## Your Turn

Experiment with a number cube:

- 1,2 - red
- 3,4 - yellow
- 5,6 - purple

### The Experiment

- Predict the outcome if you roll the number cube once.
- Roll the number cube and record the color on the Tally Sheet.



Why does the number cube work like the candies?

Did the colors turn out as you predicted?



What fraction of the time will the first candy be:

- red?
- yellow?
- purple?

Tally Sheet

Red	Yellow	Purple

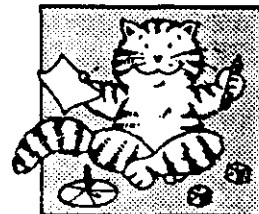
Need: 1 number cube



Save: Tally Sheet for Candy Chances (page 39).



# Candy Chances



Name \_\_\_\_\_

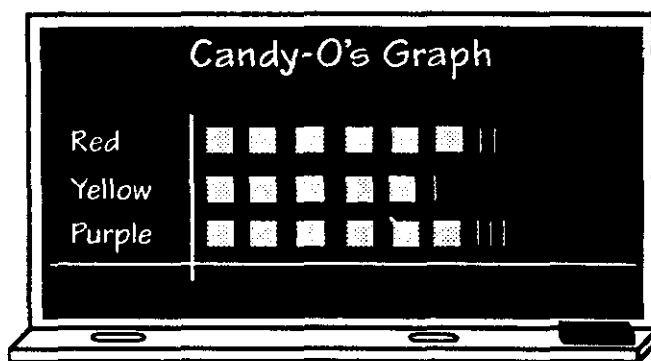


Work in a small group.

Combine the results from *Take a Candy* to develop group totals.

## Graph Activity

- Work as a class to combine the results.
- Use 1 colored square for each tally block of 5.
- Tape the squares on chart paper to make a class graph.



How many squares are there in total?

What fraction of candies are the following colors:

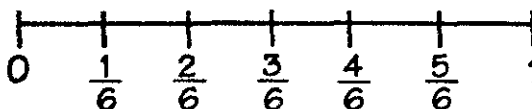
- red? • yellow? • purple?

What are the chances for each color being first in the roll?



On the scale, show the chances of each color being first:

- red (R) • yellow (Y) • purple (P)



Need: Chart paper; 2" x 2" paper squares; red, yellow, and purple data from *Take a Candy* (page 38).

## Teaching the Standard (Number Sense) With Children's Literature

\*Books dealing with this subject that could be incorporated into mathematics instruction.

Alligator Pie by Dennis Lee (Macmillan, 1987).

The Best Vacation Ever by Stuart J. Murphy (HarperCollins, 1997).

Can You Count to a Googol? by Robert Wells (Albert Whitman, 2000).

Do You Wanna Bet?: Your Chance to Find Out About Probability by Jean Cushman  
(Clarion Books, 1991).

The Doorbell Rang by Pat Hutchins (Greenwillow, 1986).

Fraction Action by Loreen Leedy (Holiday House, 1994).

Fraction Fun by David Adler (Holiday House, 1996).

The History of Counting by Denise Schmandt-Besserat (Morrow, 1999).

How Much, How Many, How Far, How Heavy, How Long, How Tall Is 1,000? by Helen  
Noland (Kids Can Press, 1995).

How Much Is a Million? by David M. Schwartz (Lorthrop, Lee, and Shepard, 1985).

If You Give a Mouse a Cookie by Laura Joffe Numeroff (Harper and Row, 1995).

Jumanji by Chris Van Allsburg (Houghton Mifflin, 1981).

The King's Commissioners by Aileen Friedman (Scholastic Press, 1995).

Let's Investigate Statistics by Marion Smoothey (Marshall Cavendish, 1993).

Lemonade For Sale by Stuart Murphy (HarperCollins, 1998).

A Million Fish...More or Less by Patricia Mc Kissack (Dragonfly, 1996).

On Beyond a Million: An amazing Math Journey by David M. Schwartz  
(Doubleday/Random House, 1999).

One Hundred Hungry Ants by Elinor J. Pinczes (Houghton Mifflin, 1993).

Pigs at Odds: Fun With Math and Games by Amy Axelrod (Simon & Schuster, 2000).

A Three Day Hat by Laura Geringer (Harper and Row, 1985).

## **Standard 2 — Computation**

Fluency in computation is essential. As students learn about the whole numbers up to 1,000, they learn how to add and subtract them. They develop the concepts of multiplication and division from addition and subtraction and learn basic multiplication and division facts. They also start to add and subtract fractions with the same denominator.

## **Standard 2**

### **Computation**

*Students solve problems involving addition and subtraction of whole numbers. They model and solve simple problems involving multiplication and division.*

3.2.1 Add and subtract whole numbers up to 1,000 with or without regrouping, using relevant properties of the number system.

Example:  $854 - 427 = ?$  Explain your method.

3.2.2 Represent the concept of multiplication as repeated addition.

Example: Lynn made 3 baskets each week for 4 weeks. Draw a picture to show how many baskets she made.

3.2.3 Represent the concept of division as repeated subtraction, equal sharing, and forming equal groups.

Example: Bob shared 10 cookies among 5 friends. Draw a picture to show how many cookies each friend got.

3.2.4 Know and use the inverse relationship between multiplication and division facts, such as  $6 \times 7 = 42$ ,  $42 \div 7 = 6$ ,  $7 \times 6 = 42$ ,  $42 \div 6 = 7$ .

Example: Find other facts related to  $8 \times 3 = 24$ .

3.2.5 Show mastery of multiplication facts for 2, 5, and 10.

Example: Know the answer to  $6 \times 5$ .

3.2.6 Add and subtract simple fractions with the same denominator.

Example: Add  $\frac{3}{8}$  and  $\frac{1}{8}$ . Explain your answer.

3.2.7 Use estimation to decide whether answers are reasonable in addition and subtraction problems.

Example: Your friend says that  $79 - 22 = 27$ . Without solving, explain why you think the answer is wrong.

3.2.8 Use mental arithmetic to add or subtract with numbers less than 100.

Example: Subtract 35 from 86 without using pencil and paper.

## Example Computation Lesson

## Lesson Plan Content Page

**Name:** Abby Land

**Lesson Topic:** Inverse Relationships Between Multiplication and Division

**INTASC Principle:** The professional educator understands content. (#1)

**IN State Standard:** Standard 2 Computation- Students solve problems involving addition and subtraction of whole numbers. They model and solve simple problems involving multiplication and division.

**IN State Indicator:** 3.2.4- Know and use the inverse relationship between multiplication and division facts, such as  $6 \times 7 = 42$ ,  $42/7 = 6$ ,  $7 \times 6 = 42$ ,  $42/6 = 7$ . *Example:* Find other facts related to  $8 \times 3 = 24$ .

### **Annotated Bibliography:**

#### **Instructional:**

Kremer, Ronald. Ready to Play: Multiplication and Division Games. Hayward, CA: Activity Resources Company, Inc., 1977.

This book provides many different ideas for games that could be used to teach multiplication and division. One game entitled "Divide/Multiply Race" contains great ideas for teaching these concepts and could easily be modified to teach the concept of inverse relationships. This book is available in the Educational Resources department of Bracken Library, which is located on the Ball State University campus.

The Super Source: Snap Cubes (Grades 3-4). New York: Cuisenaire Company of America, Inc., 1996.

This source includes a lesson entitled "Loose Caboose" that provides ideas on how to teach multiplication and division facts using snap cubes. The lesson concerns the use of array models to represent problems. This book is available in the Educational Resources department of Bracken Library, which is located on the Ball State University campus.

#### **Informational:**

Madell, Robert. Picturing Multiplication and Division: From Models to Symbols. Palo Alto, CA: Creative Publications, 1979.

This book provides information on different styles of picture representations that can be used with multiplication and division and gives examples of each type. This book is a great source of information for people who are not familiar with non-linguistic representations. This book is available in the Educational Resources department of Bracken Library, which is located on the Ball State University campus.

Thornton, Carol A., and Cathy Nixon. Looking into the Facts: Multiplication and Division (Teacher's Guide). Palo Alto, CA: Creative Publication, 1977.

This guide provides basic information about multiplication facts and their patterns to the teacher. It also provides ideas about what to emphasize when teaching each fact to

the students. This book is available in the Educational Resources department of Bracken Library, which is located on the Ball State University campus.

**Children's Literature:**

Scieska, Jon. Math Curse. New York: Viking, 1995.

This is a piece of children's literature about a little girl who realizes almost everything she does contains math. She begins to think of everything as a math problem and cannot escape from her curse. This book is available in the Educational Resources department of Bracken Library, which is located on the Ball State University campus.

## Lesson Plan

**IN State Standard:** Standard 2 Computation- Students solve problems involving addition and subtraction of whole numbers. They model and solve simple problems involving multiplication and division.

**IN State Indicator:** 3.2.4- Know and use the inverse relationship between multiplication and division facts, such as  $6 \times 7 = 42$ ,  $42/7 = 6$ ,  $7 \times 6 = 42$ ,  $42/6 = 7$ . *Example:* Find other facts related to  $8 \times 3 = 24$ .

**Lesson Objective:** The students will identify inverse relationships for multiplication and division facts after being given these facts during the inverse race board game.

**Materials/Media:** paper, pencils, inverse card pairs for motivation (one for each student), snap cubes, cards with multiplication fact for guided practice (one for each student), cards with division fact for guided practice (one for each student), cards with different facts for the check for understanding section (one for each student), game board, deck of game cards, checklist, and the book *Math Curse*.

### New Information:

- Each multiplication fact has two division facts that are its inverse. (e.g.  $4 \times 2 = 8$  has inverses  $8/2 = 4$  and  $8/4 = 2$ )
- Each division fact has two multiplication facts that are its inverse. (e.g.  $56/7 = 8$  has inverses  $8 \times 7 = 56$  and  $7 \times 8 = 56$ )

**Motivation:** Each student will be given two different cards with a math problem written on each. One of the problems will be a division problem and the other will be a multiplication problem. These problems will be inverses of each other. The multiplication problem will not have the answer provided, but the division problem will be complete with answer. The students will then be asked to solve this problem, and to see if they can find any similarities between the two problems. The students will be allowed to use any means to solve this problems. Snap cubes will be provided in case the students would like to use these to help solve the problem. Once all the students are done with their problems or the teacher can tell it is time to move on, she will ask the students, "Does anyone notice anything their two problems have in common?" The teacher and the students will then discuss the similarities in these problems. **(Identifying Similarities/Differences)** The teacher will then ask, "Do you think all problems have another problem that is similar just like these are?" Discuss this question with the students and then tell them, "Today we will be learning about facts similar to these facts, they are called inverses or fact families." **Goal for Learner**

### Procedure:

1. **New Information:** Each multiplication fact has two division facts that are its inverses, and each division fact has two multiplication facts that are its inverses. These inverse facts are sometimes called fact families.



- 2. Modeling:** The teacher will make an array model on the desk for  $5 \times 4 = 20$ . The teacher will explain that since this is a multiplication fact it has two division facts that are its inverses. The teacher will then model these two facts ( $20/4=5$  and  $20/5=4$ ) with the snap cubes. The teacher will explain to the students how she knew these problems were the two inverses. To find the inverse of a multiplication problem, you start with the answer (in this case 20) and divide it into as many groups as the first number (5) and then the answer, or number in each group, is the other number that was part of the multiplication fact (4). This will be done for both inverses. The teacher will then model the division fact  $54/6=9$  and explain that this fact has two multiplication inverses. The teacher will then model these inverses ( $6 \times 9=54$  and  $9 \times 6=54$ ) by drawing them on a paper in the form of an array model. **(Non-linguistic Representations)** The teacher will explain how to find these inverses. First the teacher will tell the students that the number being divided into groups (the largest number) will be the answer to the two multiplication problems. Then the teacher will explain that the other two numbers are going to be multiplied together to form the multiplication problem. The teacher will then show how these problems look different depending on the order of the two numbers being multiplied by referring back to the array models.
- 3. Guided Practice:** The students will then each be given a card with the same multiplication problem ( $2 \times 8 = 16$ ) written on it. The students will then be told to find the two division problems that are the inverse of this multiplication problem. The students will be encouraged to use the snap cubes to help them visualize the problem, however, the students may choose to do the problem without the snap cubes if they would like. The teacher will walk the students through the problem by having them first model the problem. Then the teacher will ask the students what number they should start with as the first number of their division problems. If the students answer correctly they will move on if not the teacher will instruct further. Then the teacher will ask them what numbers they could divide by and what their answers would be. The students will then try each of these problems by splitting their snap cubes into however many groups they are dividing by to see if the number left in each group was what they expected. The students will then all be given a card with the division fact  $35/7=5$  on it. The students will be instructed to try and find the two multiplication inverses to this problem without using snap cubes. The teacher will walk the students through the steps by first asking them what number will be the answer to the problem. The teacher will then ask the students to draw this problem on their paper using squares as if they were drawing the snap cube problem. **(Non-linguistic Representations)** The teacher will assist the students if they have any problems.
- 4. Check for Understanding:** Ask the students, "How many inverses does a multiplication or division problem have? What type of a problem is the inverse of a multiplication problem? What type of a problem is the inverse of a division problem?" **(Questions, Cues, Advance Organizers)** Then give

each student one card with a multiplication or division problem written on it (complete with answer). Tell the students to find the inverses of the problems on their card. Allow the students to draw these facts on paper if they would like. **(Non-linguistic Representations)** If the students have problems the teacher will go back over the information.

5. **Practice/Application:** The students will play an inverse race board game. Each student will be given a different color marker (snap cube), and the student with the closest birthday will be allowed to go first. The student will place their marker on the start and draw a fact out of the pile. If the student can name a correct inverse of the fact they will move forward the number of spaces written on the card. If the student does not name a correct inverse of the fact they will stay where they were. Then the students will take turns drawing cards (the game will move in a clockwise order). The game will end once one student reaches the finish.
6. **Closure:** The teacher will ask the students, "What did you learn during today's lesson?" The teacher and the students will discuss the answers. Then the teacher will ask the students, "How could knowledge of inverses help you with your math facts?" The teacher and students will discuss possible responses to this question. **(Questions, Cues, Advance Organizers)**

**Evaluation of Student Learning:** The teacher will have a checklist with all the students' names on it that they will use while the students play the game. Next to the student's name will be three columns. One will be used to place tally marks for each problem the student answers correctly, another to make tally marks for each problem the student answers incorrectly, and the third will be for teacher comments.

**Lesson Extension:** If there is time left over, the teacher will read the book *Math Curse* to the students. The teacher will use the book as a way of showing students how often math is involved in their everyday life. To further the students' understanding of inverse relationships, the teacher will stop at points in the book, which have been marked. At these points the teacher will ask the students a multiplication or division problem that goes along with the book and then ask the students to find its inverse.

## Evaluation Checklist

[illegible]

## More Teaching Ideas for Computation

## Teaching Ideas Bibliography (Computation)

Bloomer, Anne M., and Phyllis A. T. Carlson. Activity Math: Using Manipulatives in the Classroom. Menlo Park, CA: Addison-Wesley Publishing Company, 1993.

\*Contains "Multiplication 1" and "Division 1" activities on pages 145 and 146, and 171 and 172.

Bresser, Rusty, and Caren Holtzman. Developing Number Sense (Grades 3-6). Sausalito, CA: Math Solutions Publications, 1999.

\*Contains "Oh No! 99!" on pages 17, 18, 19, and 20.

Greenburg, Dan. Funny and Fabulous Fraction Stories: 30 Reproducible Math Tales and Problems to Reinforce Important Fraction Skills. New York: Scholastic Professional Books, 1996.

\*Contains "Rex Roper's Believe It or Not!" story on pages 28 and 29.

The Super Source: Snap Cubes (Grades 3-4). White Plains, NY: Cuisenaire Company of America, Inc., 1996.

\*Contains the "Loose Caboose" and "Train and Boxcars" ideas on pages 34, 35, 36, and 37, and 78, 79, 80, and 81.

# TRAINS AND BOXCARS

NUMBER

- Multiplication
- Addition

## Getting Ready

### What You'll Need

Snap Cubes, about 40 per group

Dice, 1 die per group

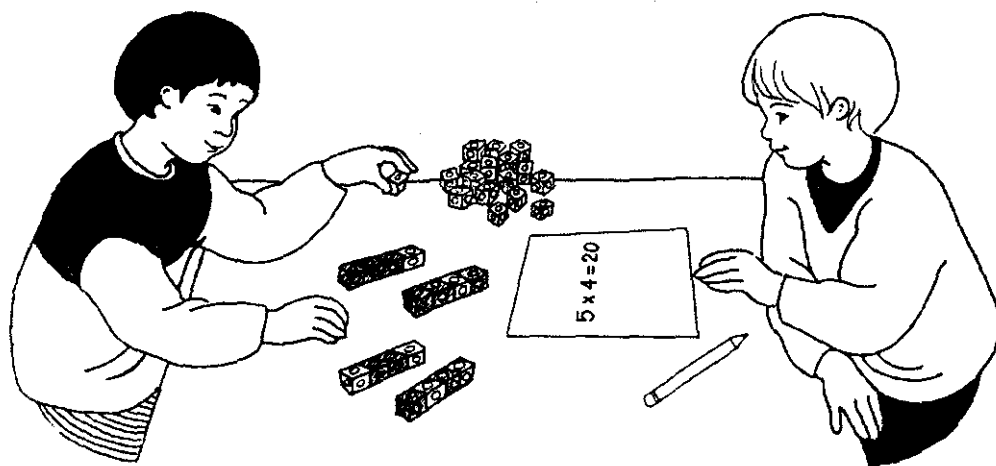
Snap Cube grid paper, page 90

Calculators, 1 per group (optional)

## Overview

Children play a game in which they roll a die to determine the number of Snap Cubes to put in a train. They roll the die a second time to determine the number of trains to make. In this activity, children have the opportunity to:

- ♦ view multiplication as repeated addition
- ♦ practice using multiplication symbolism correctly
- ♦ add long columns of numbers



## The Activity

*You may have to explain what a boxcar is.*

*You may wish to explain that  $4 \times 3 = 12$  is shorthand for saying 4 trains of 3 boxcars equals 12 boxcars.*

## Introducing

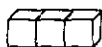
- ♦ Tell children that you are going to play a game in which Snap Cubes represent boxcars.
- ♦ Ask for three volunteers. One will be your partner and the other two will be the team you are playing against.
- ♦ Ask your partner to roll the die. The number that comes up tells how many boxcars are on the train. Have your partner take that number of Snap Cubes and build a train.
- ♦ Roll the die again to determine how many of those trains to make.
- ♦ On the chalkboard, record a picture of the trains and the multiplication fact to go with it. For example, if you rolled a 3 and then a 4, the picture would show 4 trains of 3 boxcars, and the multiplication fact would be  $4 \times 3 = 12$ .
- ♦ Let the other team play the game and record their trains and multiplication fact on the board.
- ♦ Compare the two products.

## On Their Own

### Play Trains and Boxcars!

Here are the rules:

1. This is a game for 2 teams of 2 players each. In this game, a boxcar is 1 Snap Cube. The object of the game is to be the team with more boxcars.
2. The first team rolls the die to find out how many boxcars to put on a train. The team builds a Snap Cube train with that number of boxcars.



A train with 3 boxcars

3. The first team rolls the die again to find out how many of those trains to build and then builds the rest of the trains, if necessary.
  4. After all the trains are built, the first team draws a picture of the trains and records the multiplication fact that tells how many boxcars there are in all.
  5. Then the second team takes their turn.
  6. Play continues until each team has had 7 turns.
  7. Each team finds the sum of all the boxcars from the 7 rounds. The team with the greater number wins.
- Save your recording sheets for class discussion.

## The Bigger Picture

### Thinking and Sharing

Call children together with their recording sheets to discuss the mathematics.

Use prompts such as these to promote class discussion:

- ♦ What is the smallest number of boxcars you could get in one turn? How could you get that?
- ♦ What is the greatest number? How could you get that?
- ♦ Did any team get 4 (10, 16, 20) boxcars in one turn? How did you get that many?
- ♦ Did anyone get any odd numbers of boxcars in one turn? How did that happen?
- ♦ What are the possible numbers of boxcars you can get in one turn? Explain.

### Writing

Ask children to write a description of all the ways they could get 12 boxcars in one turn of play.

### Extending the Activity

1. Have children play the game using non-standard dice. You might affix stickers to each side of a die with larger numbers written on them. You could also use tetrahedra dice (4-sided) or decahedra dice (10-sided).

## Teacher Talk

### Where's the Mathematics?

This activity provides an introduction to the concept of multiplication as repeated addition. Children link pictures of the concrete manipulatives with the symbolic way to represent this repeated addition with a multiplication equation.

Children will find that the smallest number of boxcars is 1 ( $1 \times 1$ ) and the largest number is 36 ( $6 \times 6$ ), but not every number between 1 and 36 is a possible product. This list shows all of the possible outcomes:

ROLL OF DICE	PRODUCT	ROLL OF DICE	PRODUCT
1, 1	1	4, 1	4
1, 2	2	4, 2	8
1, 3	3	4, 3	12
1, 4	4	4, 4	16
1, 5	5	4, 5	20
1, 6	6	4, 6	24
2, 1	2	5, 1	5
2, 2	4	5, 2	10
2, 3	6	5, 3	15
2, 4	8	5, 4	20
2, 5	10	5, 5	25
2, 6	12	5, 6	30
3, 1	3	6, 1	6
3, 2	6	6, 2	12
3, 3	9	6, 3	18
3, 4	12	6, 4	24
3, 5	15	6, 5	30
3, 6	18	6, 6	36

The possible products, listed in order from smallest to greatest are: 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, 30, and 36.

When asked if every number between 1 and 36 is a possible product, children probably will not think to make the exhaustive list shown above. Some



2. Pool all of the class' products and make a frequency distribution graph of the products that came up in one game.

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children may mentally consider a list of products from 1 through 36 and think about whether each of the numbers on that list could be a possibility. After some thought, they are apt to respond, "7 isn't possible" or "17 isn't possible." Other children may suggest making a tally list by asking, "If you got a 7, raise your hand" and continue down the list of possible products from 1 through 36 this way.

Using the data the children collected while playing the game and exploring the different ways some products, such as 4, 10, 16, and 20, could arise touches upon probability and also contributes to building number sense. For example, 4 could arise as 2 trains of 2 boxcars or 1 train of 4 boxcars or 4 trains of 1 boxcar while 16 could only happen one way: 4 trains of 4 boxcars. Looking at how the order of the numbers changes from 1 train of 4 boxcars to 4 trains of 1 boxcar while the product stays the same gives children experience with the Commutative Property of Multiplication.

*This game can be a worthwhile activity to leave in a math center so children may return to it later. It also can be suggested to parents, who ask, "What can we do to help our child in math?" When playing the game at home, manipulatives, such as lima beans instead of Snap Cubes, could be suggested.*

Children will probably notice that even products occur more often than odd products. Some children may look for patterns to explain why this happens and realize that:

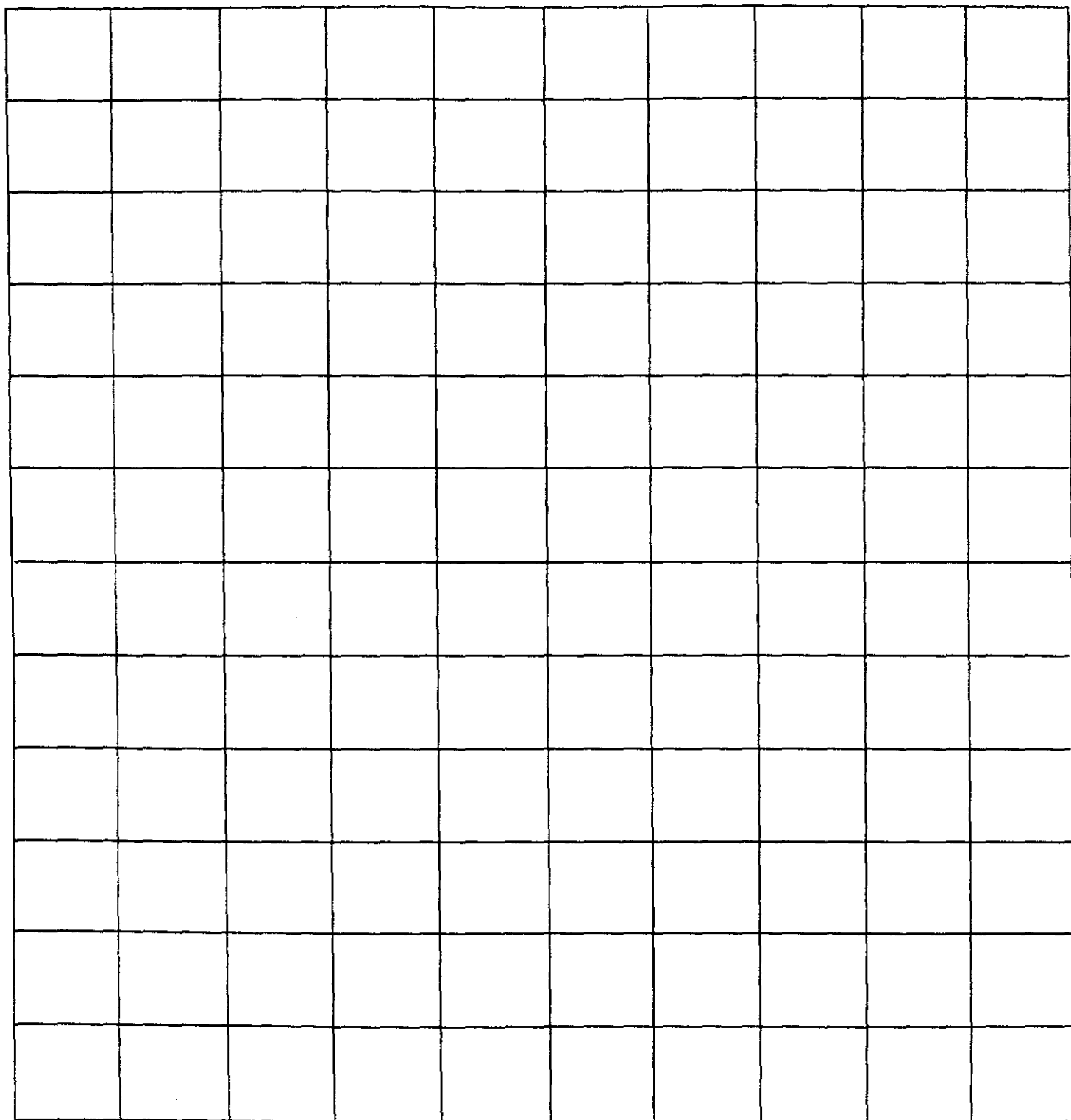
even x even	=	even product
even x odd	=	even product
odd x odd	=	odd product
odd x even	=	even product

Since each die has 3 even numbers (2, 4, 6) and 3 odd numbers (1, 3, 5), the chances of getting an even or odd number are the same, and so the odd products will only show up one-quarter of the time. The list of the possible products on the preceding page shows that odd products only show up 9 times out of 36, or one-quarter of the time, as predicted. This analysis of even and odd products may help some children when they memorize their multiplication fact tables.

Spending plenty of instructional time on the concept of multiplication will pay off later when children have a mental image of  $4 \times 3$  and the ability to reconstruct this fact as repeatedly adding 4 trains of 3 boxcars.

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## SNAP CUBE GRID PAPER



# Multiplication 1

**Skill:** To understand the concept of multiplication using serial addition

**Time:** 1 period

**Materials:**

*beads and string:* 3 or 4 of each color available per student pair

*treasure box or other objects*

*geoboards and rubber bands*

See Reproducibles for: geoboard paper—1 sheet per student, graph paper with 1" squares—1 sheet per group

## Anticipatory Set

Use *beads and string*.

Have students work in pairs and sort out three groups of two beads each. How many groups do they have? (3) How many beads are in each group? (2)

Have them take three beads of each color and place them on their desks so that they can see them all. How many groups did they have? (Answers will depend on how many different colors were available.) How many beads were in each group? (3)

Next have students string necklaces of groups of beads, such as three red beads, three clear beads, three green beads, three blue beads, and three yellow beads. How many beads did it take to complete the necklace? (15) Encourage students to generate similar situations and arrive at total amounts.

Students continue to use the manipulatives to represent small groups of small numbers until they are familiar with the properties of serial addition problems.

**Now give the students some examples of unequal groups. Example: donations of 5 bags of clothes; in one bag were 4 items, another had 6 items, and so on. What is the difference with this type of addition? (The numbers are not all the same.)**

Students discuss the necessary properties for serial addition problems and practice with small groups of numbers until they are confident about the process.

**Explain that when we add equal groups as we have been doing in serial addition, a shortcut called *multiplication* may be taken. That process is one we will begin to learn.**

## Procedure 1

To understand serial addition

Use *objects or beads and string*.

**Have students work in pairs or groups of three or four. Make sure that each group has enough objects for some practice in serial addition. Have students set out four groups of three objects each. Then have them work together to arrive at a total. Did they count or add?**

Students discuss their findings and make up situations in which they might use the idea of serial addition.

**Have students take beads of different colors or objects of different kinds and place them into groups of two or three each. Ask: How many groups do you have? How many objects are in each group?**

Students discuss what they did. They report how many groups of objects, how many were in each group, and how many total objects they had. They should become aware of adding groups to arrive at total amounts.

**Give students many groups of equal numbers that they need to total; e.g., donations to the Goodwill of 6 bags of 7 shirts each; how many shirts in all?**

## Procedure 2

To develop the concept of multiplication as serial addition

Use *geoboards, rubber bands, and geoboard paper*.

**Have students work in groups with one geoboard per group and one sheet of paper per student. They should take a rubber band and close off a figure that is one square by two squares, or two pins by three pins.**

**Have them enclose another figure of the same size and shape.**

Students complete the task and copy the figures onto their geoboard paper.

**Ask: How many squares were enclosed in all? (4) Did you add or count?**

**Put on the board 2 groups of 2 squares = 4 squares. This is the form for multiplication. Write  $2 \times 2 = 4$ . This is math language for this activity.**

**Have students enclose one more figure like the first. Now how many squares in all? (6) Put  $3 \times 2 = 6$  on the board.**

Students copy the third figure onto their geoboard paper.

**Continue the procedure with other-size rectangles. Write each discovered fact on the board.**

When students become comfortable with the process, you can move to the stage at which students write, as well as use manipulatives.

As students complete each figure on their geoboards or paper, have them copy the equation that you have written on the board onto their geoboard paper below the figure represented.

It is important that students understand that the written form is just a description of what is actually happening.

Have students work together to make other problems on the geoboards and copy onto their paper, counting to discover the multiplication sentence proven. They are not actually trying to solve problems but are writing down what their geoboards show.

Have each student write down a problem situation in which each of several people have the same number of some object. They exchange with a peer to solve and then post the completed problems as samples of serial addition.

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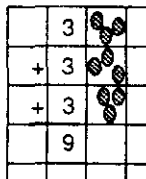
### Procedure 3



To practice serial addition

Use *any objects* and *graph paper*.

Have groups of three or four students use the objects on graph paper to show problems involving serial addition and place the serial addition solutions beside the objects.



For example, this graph could show the situation of three students with three erasers each. How many erasers in all?

Students work in groups to make up similar situations, to display objects, and to write serial additions.

In the next stage, have students write and solve problems first, then check with manipulatives.

Continue until they grasp the concept of multiplication as serial addition and are ready to move on to some basic facts.

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### Practice and Extension

Students use *any of the objects* and write problems that can be solved with serial addition. They answer the problems on the back of the paper and then exchange papers with a partner.

Have students make up people problems and then act them out: If three pairs of students work together, how many students in all?

# LOOSE CABOOSE

NUMBER

- Division
- Multiplication

## Getting Ready

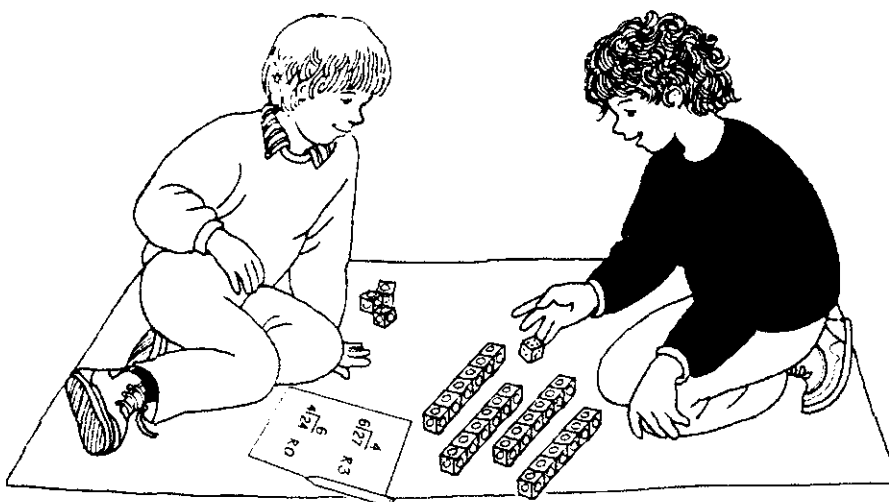
### What You'll Need

Snap Cubes, 27 per pair  
Dice, 1 die per pair

## Overview

Children play a game in which they roll a die to determine how many trains of equal length to build from a pile of 27 Snap Cubes. They write a division sentence to describe what happens during each turn. In this activity, children have the opportunity to:

- ♦ view division as making same-size sets
- ♦ practice using division symbolism
- ♦ look for patterns in division problems



## The Activity

*You may have to explain that the caboose is the last car on a freight train.*

*You may want to play a game of Loose Caboose with children before they begin On Their Own.*

## Introducing

- ♦ Show children a pile of 17 Snap Cubes. On the chalkboard, write  $\overline{)17}$ .
- ♦ Ask a volunteer to roll a die. Use the number that comes up to determine how many trains of equal length to build. Build that number of trains and set aside the remaining cubes, or the "loose caboose." For example, if the volunteer rolls a 3, form 3 trains of cubes each and set aside the 2 leftover cubes.
- ♦ On the chalkboard, finish the division problem  $3 \overline{)17} R2$ . Identify 3 as the number of trains, 5 as the number of Snap Cubes in each train, and 2 as the remaining number of Snap Cubes, or the loose caboose.
- ♦ Now set aside the 2 loose caboose cubes and use the Snap Cubes that are left to write a new division:  $\overline{)15}$ . Have another volunteer roll the die and repeat the activity.

## On Their Own

### Play Loose Caboose!

Here are the rules.

1. This is a game for 2 players. The object is to wind up with more Snap Cubes.
2. Start with a pile of 27 Snap Cubes. Decide who goes first.
3. The first player writes the beginning of a division problem,  $\overline{)27}$ , and rolls the die to find out how many trains of equal length to build from the 27 cubes.
4. The first player builds the trains and keeps any "loose caboose" cubes that are left after the trains are built. Each of the trains should be as long as possible.
5. The first player completes the division problem. For example, if a 4 was rolled:

$$\begin{array}{r} 6 \leftarrow \text{Number of cubes in each train} \\ \text{Number of trains} \rightarrow 4 \overline{)27} \quad R3 \leftarrow \text{Number of loose caboose cubes} \end{array}$$

6. If there are no loose caboose cubes, the player still completes the division.
  7. The second player begins his or her turn using the cubes that are left. In the example above, there were 3 loose caboose cubes, so the second player would begin with 24 cubes and write  $\overline{)24}$ .
  8. Players take turns until there are no Snap Cubes left.
- Play at least 2 full games of Loose Caboose.
  - Look for patterns in the division problems.

## The Bigger Picture

### Thinking and Sharing

Have children post their division problems with a remainder of zero in one column. Do the same for division problems with remainders of 1, 2, 3, 4, and 5.

Use prompts such as these to promote class discussion:

- ♦ What patterns did you notice?
- ♦ Which numbers could you make into 2 trains with no leftover cubes? Into 3 trains? 4 trains? 5 trains? 6 trains?
- ♦ What happened when you rolled a 1?
- ♦ Which numbers always had leftover cubes unless a 1 was rolled?
- ♦ Which numbers had the greatest number of ways to get a remainder of zero? What happened in the game when these numbers came up?
- ♦ Which numbers and roll of the die would give you the greatest number of loose caboose cubes in one turn?

### *Drawing and Writing*

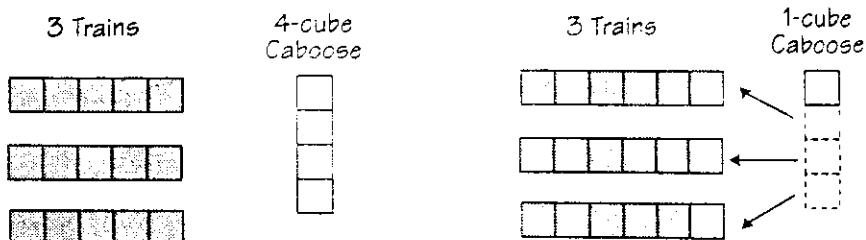
Ask children to write or draw a description of all the ways 24 Snap Cubes could be divided into trains of equal length.

## Teacher Talk

### Where's the Mathematics?

This activity provides an introduction to the concept of division as the partitioning of a set into equal-sized groups. It also conveys the meaning of the remainder. Multiplication facts are reinforced when the children start the next round of play and have to determine how many Snap Cubes are left so they can begin their division problem.

Children are likely to note that 1 is a divisor of every number; in other words, dividing by 1 always leaves a remainder of zero. Children are also likely to point out that the remainder (the number of cubes in the loose caboose) is always less than the divisor (the number of trains). Children can verify this by examining the trains and the number of caboose cubes. If the number of cubes in the loose caboose is equal to or greater than the number of trains, then each of the trains can be made longer.



Children will notice that the numbers 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, and 26 can be made into two trains of equal length with no remainders. They will recognize these numbers as even numbers and perhaps supply their own definition of even numbers as "the doubles" or "numbers that make equal trains with no leftovers when you roll a 2."

When they look at the numbers that have 3 as a divisor and no remainder—namely, 3, 6, 9, 12, 15, 18, 21, 24 and 27—children will be reminded of the multiplication table for 3. They may notice that this list includes every third number from the list of even numbers and has an odd-even pattern.

## *Extending the Activity*

Have children play the game with one of these variations:

- Use an 8-sided or 10-sided die.
  - Use a standard die labeled with larger numbers.
  - Start with a different number of Snap Cubes.
- 

Four is a divisor for 4, 8, 12, 16, 20, and 24. These numbers are made up of every other number from the even list. As the size of the divisor increases, the list of numbers shortens. By the time children get to the numbers that have 5 as a divisor, there are only five: 5, 10, 15, 20, and 25. The list for 6 is even shorter with only 4 numbers: 6, 12, 18, and 24.

Children are apt to report that 12 and 24 are the numbers they got “stuck on” in the game. By this, they mean that any roll of the die, except 5, resulted in trains with an equal number of cars and no leftover cabooses; so play went back and forth between the two players with no one winning any loose caboose cubes until a 5 was rolled.

When the children look for numbers that always give loose caboose cubes (unless a 1 was rolled), they will find 7, 11, 13, 17, 19, and 23. Later, children will learn that these numbers are part of the set called prime numbers, which have exactly two divisors: the number itself and 1. The prime numbers 2, 3, and 5 would not belong on the children’s list because these numbers have no leftovers when the numbers themselves (2, 3, or 5) are rolled.

By a lucky roll of the die or if the children have played the game enough to be able to compile exhaustive lists of data, they will see that the numbers 11, 17, and 23 hold the potential for rolling a 6 on the die and netting the largest number (5) of loose caboose cubes possible for this game.

In analyzing the parts of their division problems — namely, the dividend, divisor, quotient, and remainder — children get their first taste of the study of number theory and a foundation for dealing with division in an algebraic context.

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## Division 1

**Skill:** To learn division concepts: sharing and repeated subtraction

**Time:** 1 week

**Materials:**

*similar objects for sharing:* beans, tiles, counters, etc.; about 20 per student

*containers, or 6" × 6" squares of paper:* 1 per student  
*graph paper:* 1/2" or larger squares  
*markers*

*interlocking cubes, or Cuisenaire® rods*

*geoboards and rubber bands*

See Reproducibles for: hundred boards—1 per student

### Anticipatory Set

Use *similar objects for sharing*.

Have three students stand up. Say: I have a number of beans I want to share. How can I do it?

Discuss sharing with students. Reinforce that objects are to be shared equally among the three.

### Procedure 1

To develop the concept of division as sharing

Use *objects for sharing and containers or squares of paper*.

Give each student about 20 beans. Vary the amounts. Say: These are magic beans. You have four friends you want to give them to. Find a way to give each friend the same amount of beans. If you have any left over, just put them to the side.

Students place beans in containers or on paper squares.

Ask each student: How many beans did you start with? How many beans did each friend get? Did each get the same amount? How many beans did you have left over?

Restate the process for each student: You shared your beans evenly with four friends. We can say you *divided* your beans evenly into four piles. Each friend or pile got five beans (if students started with 20).

Repeat the activity several times, having each student use a different number of beans.

Repeat the questions and the restatement of sharing as division as before.

What did you notice when you used fewer beans for your dividing? What did you notice when you used more beans?

Students respond: More beans meant the friends each got more. Fewer beans meant they got fewer. Some numbers of beans had a lot of leftovers, and some had none.

Have students look at the last results of their sharing. For example: We have four piles with five

beans in each pile. That's 20 beans we shared. Do the results look like anything we've done before? How could I write this in math language?

Students respond: It looks like multiplication. We could write  $4 \times 5 = 20$ .

Repeat the procedure with differing amounts of beans and friends, comparing and discussing results. Students can make up different stories to go with each problem.

Go on to Procedure 2.

### Procedure 2

To record division as sharing; to introduce the  $\div$  symbol

Use *objects for sharing and containers or squares of paper*.

Have each student start with 20 beans and divide them evenly among three friends.

Say: We'll want to keep a record of our results, and look for any patterns.

Draw the following chart headings on the board:

#Beans		#Friends	#For Each Friend	#Left
21	+	3	7	0
20	+	3	6	2
19	+	3	6	1

Say: We use the division symbol,  $\div$ , to show sharing evenly. It looks like two beans shared evenly, one above and one below a line.

Vary the number of beans shared among three friends, and record the results as shown above. What do students notice?

Students should notice that a larger starting number gives a larger amount in each group. They may notice that three are never left over.

What would happen if you had three left over?

Students respond: Enough would be left for another sharing.

Can students find multiplication on the chart? Discuss the multiplication/division relationship.

Repeat the procedure with a different number of friends, and have students predict results, chart, and discuss.

Now have students record each division experiment on paper as shown:

$$20 \div 4 = 5 \quad \text{and} \quad 0 \text{ left over} \quad 5 \times 4 = 20$$

$$19 \div 4 = 4 \quad \text{and} \quad 3 \text{ left over} \quad 4 \times 4 = 16$$

$$18 \div 4 = 4 \quad \text{and} \quad 2 \text{ left over} \quad 4 \times 4 = 16$$

Continue until students are comfortable with division as sharing and the relationship to multiplication. Emphasize that the written form shows what is actually happening.

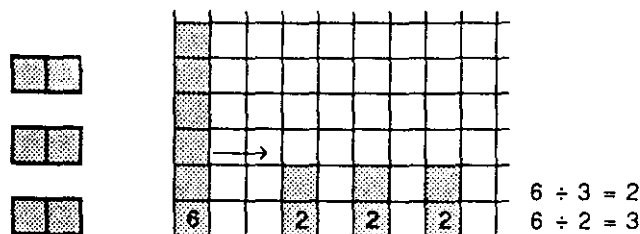
Have students make up stories to go with each experiment: fire extinguishers shared among four dragons, mud puddles shared among five pigs, etc.

### Procedure 3

To practice and record division as sharing

Use *graph paper*, *markers*, and *interlocking cubes or Cuisenaire® rods*.

Follow the same basic procedure as before. When using the interlocking cubes, have students work in pairs. One student divides the cubes, and the other records as shown:



When the division is completed, have students make cubes into sticks. How many are in each stick? How many are shown in each column on paper?

Students complete the task. They compare results and then write them on paper as shown in Procedure 2.

If using Cuisenaire® rods, students make a starting number and then use an equal number of one rods for the division. After all rods have been shared, they find an equivalent rod for each group, charting as before.

### Procedure 4

To develop the concept of division as repeated subtraction; to relate to multiplication

Use *objects for sharing* and *containers or squares of paper*.

Say: You have been sharing magic beans with friends. If you were dividing the beans among three friends, you gave one bean to each of the friends and then started over. Each time, you took three from your total beans so that you could share.

If we have lots of beans and lots of friends, sometimes there's a faster way to divide.

We could take out enough at a time for each person to get one. Let's share among three people. Take out enough beans for each person to have one.

Students do this.

Ask: How many did you take out? (3) Take out another group and put it into another pile for the second sharing.

Students do this.

How many out this time? (3 again) How many piles so far? (2 for 2 sharings) So each friend will get how many so far? (2)

Repeat until no more groups can be removed. Any leftovers?

Students respond, depending on how many beans they started with.

We'll save those left over for another time because there's not enough for each friend to have the same amount.

Now, notice how many piles you took out of your starting number, and then share each pile among your three friends.

Students share each pile.

How many groups did you start with? (answers vary) How many beans did each friend get?

Students respond: Each friend got one bean from each pile.

Repeat with a variety of numbers. How do you know what size group to subtract from your total each time?

Students respond: You need enough for each friend to get one. The size of the group is the same as the number of friends.

Discuss that to divide 20 beans among four friends, we can subtract groups of four. Division is a shortcut for subtracting the same number each time. The answer is how many times we took a group out.

Relate this to multiplication. How could we know how many beans we started with?

Students respond: Add up the beans in each group. Some will say: Multiply the number of beans times the number of groups.

Repeat until students are comfortable with the idea of division as repeated subtraction.

Go on to Procedure 5.

## Procedure 5

To record division as repeated subtraction

Use *objects for sharing* and *containers or squares of paper*.

Explain that we need to keep track of our results and look for patterns. Write the following chart headings on the board.

# Beans	# Friends	Group Size	# Groups	# Each Gets	# Left
21	÷	3	7	7	0
20	÷	3	6	6	2
19	÷	3	6	6	1

Students do division as you keep the chart. They discuss all patterns and the relationship of multiplication to division, as in Procedure 4.

When students are comfortable with charting, have them work in pairs. One should do the division and one the recording. Can they predict the results? Have them make up different stories to go with the problems.

## Procedure 6

To practice and record division as repeated subtraction

Use *geoboards and rubber bands*, *interlocking cubes*, *Cuisenaire® rods*, or *hundred boards and markers*.

If using geoboards, have students enclose the total number with a rubber band and use other rubber bands to enclose groups.

If using Cuisenaire® rods, have students make a train equal to their starting number; for example, 16. If they are dividing by four, they will lay four rods alongside until they have as many as possible. One rod is used to make up the difference, the leftovers.

If using interlocking cubes, put out enough cubes to equal the starting number. Have students subtract out enough cubes for each group and snap them together.

If using hundred boards, have students either use a marker to put a line at the end of their starting number or put a counter on it. If the number is 21, students would draw a line after 21. If they are dividing 21 by 3, they should color three spaces and then outline that group of three in another color. Next they should color a second group of three and outline that group.

When they are done making groups of three, have them count the number of outlined groups. Any numbers not colored will be the leftovers.

Students continue charting and writing results.

It is important for students to practice with a variety of materials. This helps build understanding of underlying concepts.

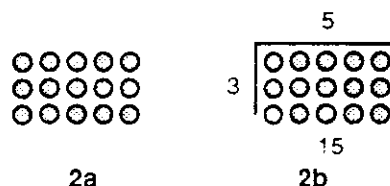
Students complete the assignment and discuss the differing answers that were found.

## Procedure 7

To introduce arrays and the  $\overline{)}$  symbol

Use *cubes*, *counters*, or *tiles and paper*.

Have students lay out 15 counters as in Illustration 2a.



Explain that the lines across are called *rows*, like a row of seats in an auditorium. The lines up and down are called *columns*, like columns holding up a building. Do students have other memory aids?

Students practice building arrays of specified rows and columns.

Now have students put out five columns with three in each column and write a 15 below as shown in Illustration 2b.

These are a farmer's 15 pigs. He has them in three lines. However, he doesn't want them to get away, so he puts a fence around them. Draw a fence as shown.

The farmer walks around to the front of the lines. (Show on the board where the three will go.) How many lines does the farmer see? (3) Write the three in the correct place. The pigs have been divided into three lines, or groups.

The farmer needs to know how many pigs are in each line, so he walks around to see. Indicate on the board where the five will go.

How many pigs in each line? (5) Write the five in place. Five pigs are in each line.

This fence is used as one of our division symbols. It shows 15 pigs divided into three groups with 5 in a group. You could also think of the problem as 15 pigs divided into five columns with 3 in each column.

Have students make arrays using other numbers of counters. Write the array and written division problem on the board as before.

Any remainders can be put to the side as left-overs, or you may write them as remainders.

**Have we done anything with rectangles or arrays before?**

Students respond: This is like multiplication. So many groups, with so many in each group, tells how many in all.

**Provide much practice for students.**

When students are comfortable with arrays, they work in pairs. One student works with the counters while the other writes the problem. As each step is done with the counters, the result is put onto paper. They write the related multiplication fact and make up stories to go along with their manipulations.

Have students pool their money and work in pairs. They are to take the \$30 and divide it into equal groups by spending an equal amount of money at each store. How much do they spend at each store, and how many stores do they visit before all their money is gone? (Answers will vary.)

Have students use *any of the materials* listed or *place value materials* to create story problems for each other to solve. Challenge students to discover ways that repeated subtraction and repeated addition are related activities.

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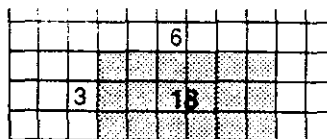
## Procedure 8



To practice the concept and writing of division, using arrays

Use *graph paper and markers* or *geoboards and rubber bands*.

Have students outline a rectangular array of their starting number on graph paper. For example, if the starting number is 18 and it is to be divided by three, students will make a rectangle three squares high.



$$18 \div 3 = 6$$

$$3 \times 6 = 18$$

$$18 \div 6 = 3$$

$$6 \times 3 = 18$$

Have students write the resulting number of columns above the rectangle and then write the division and multiplication facts discovered.

Or, have students use geoboards instead of graph paper to make their arrays. Have them use other rubber bands to enclose the columns.

Students continue to use the division symbol and write down each step as they do it.

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## Practice and Extension

Have students work individually. Give each student 15 \$1 bills. The mall is having Dollar Days. Each item they find at the mall costs \$3, so they will be dividing their money into groups of \$3. They are to keep a record on paper of each purchase and the amount left. Have students complete the exercise, continuing to subtract until they have no money left. How many items were they able to buy? (5)